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Stochastic Gravity

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We give a summary of the status of current research in stochastic semiclassical gravity and suggest directions for further investigations. This theory generalizes the semiclassical Einstein equation to an Einstein–Langevin equation with a stochastic source term arising from the fluctuations of the energy-momentum tensor of quantum fields. We mention recent efforts in applying this theory to the study of black hole fluctuation and backreaction problems, linear response of hot flat space, and structure formation in inflationary cosmology. To explore the physical meaning and implications of this stochastic regime in relation to both classical and quantum gravity, we find it useful to take the view that semiclassical gravity is mesoscopic physics and that general relativity is the hydrodynamic limit of certain spacetime quantum substructures. We view the classical spacetime depicted by general relativity as a collective state and the metric or connection functions as collective variables. Three basic issues—stochasticity, collectivity, correlations—and three processes—dissipation, fluctuations, decoherence—underscore the transformation from quantum microstructure and interaction to the emergence of classical macrostructure and dynamics. We discuss ways to probe into the high-energy activity from below and make two suggestions: via effective field theory and the correlation hierarchy. We discuss how stochastic behavior at low energy in an effective theory and how correlation noise associated with coarse-grained higher correlation functions in an interacting quantum field could carry nontrivial information about the high-energy sector. Finally, we describe processes deemed important at the Planck scale, including tunneling and pair creation, wave scattering in random geometry, growth of fluctuations and forms, Planck-scale resonance states, and spacetime foams.

1. INTRODUCTION

1.1. Semiclassical Gravity from a Quantum Open System Viewpoint

Starting from the well-cultivated and familiar terrain of quantum field theory in curved spacetime [1] in our search into deeper structures beyond semiclassical gravity with focus on the backreaction problem [2], we came

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to a crossroad 10 years ago. Having understood the real physical meaning of dissipation in the effective dynamics of spacetime generated by the backreaction of particle creation [3] with the help of the Schwinger–Keldysh closed-time-path formalism [4], we began to turn our attention to possible existence of fluctuations generated by these processes. Following the dicta of nonequilibrium statistical mechanics, we proposed to view semiclassical gravity as a quantum open system [5]. The discrepancies which exist between the matter and gravity sectors (e.g., the heavy Planck mass which allows a Born–Oppenheimer approximation to be taken in the transition of quantum cosmology to semiclassical gravity [6]) enable one to treat classical spacetime as the 'system' of interest and quantum matter field as the 'environment' in the Langevin sense (e.g., refs. 7).

This then prompted us to take a closer look at the influence functional approach [8] to the quantum Brownian model [9] because we are interested in a formalism which keeps manifest self-consistency in treating the backreaction of the environment on the system, especially on the relation between dissipation, fluctuations, noise, and decoherence [10], the latter being a central issue in the investigation of the transition from quantum to classical [11, 12]. Two sets of relations were of interest to us: The first set is between dissipation and fluctuations. The second set is between noise and decoherence. The fluctuation-dissipation relation is of course well known [13], but it is usually assumed to be valid for systems at or close to equilibrium, and in fact usually derived with linear response theory (e.g., ref. 14). It would be easy to extract the fluctuations from the dissipation if such a relation holds also for ostensibly nonequilibrium systems as in a cosmological backreaction problem, i.e., between classical dynamical spacetimes and evolving quantum fields. Whether such a relation exists in semiclassical gravity is another crucial question asked in ref. 5. If so, what is the nature of such noises? We posited that such a relation should also exist in nonequilibrium systems such as that encountered in particle creation in a dynamical gravitational field. Our reasoning was that such a relation comes about as a relation between two subsystems—after one is being coarse-grained into an environment—which when traced back should reflect the unitarity condition for the dynamics of the original closed system. With this, we can then associate fluctuations or noise in the quantum field with dissipation in the spacetime dynamics, and since dissipation is generally nonlocal, we asserted that the noise generated in particle creation would generally be multiplicative and colored. These conjectures were realized in later investigations.

1.2. Dissipation, Fluctuations, Noise, and Decoherence

In the same time frame when these questions about dissipation and noise were investigated by the author, the issue of decoherence of quantum systems and the emergence of classicality was pursued by a number of researchers coming from different backgrounds in the 1980s, using statistical mechanical concepts and methods. Specially relevant to our subject matter is decoherence in quantum cosmology and the semiclassical gravity limit [6]. The source of noise and the role it plays in this context were important issues. This brings in the second pair of relations mentioned above, that between noise and decoherence. Such a relation was suggested in ref. 10 using the Brownian motion model and Langevin dynamics as a guide. Independently, Gell-Mann and Hartle [15] in an excellent paper discussed how noise is instrumental to the emergence of classical equations of motion from quantum dynamics and how it regulates the stability of classical structures.

On the technical level, the above evolution and linkage of concepts on dissipation, noise, and decoherence was facilitated by the closed-time-path, influence functional, and decoherence functional formalisms. Just as the Schwinger–Keldysh effective action [4] enabled us to get a real and causal equation of motion [3], and the Feynman–Vernon influence functional [8] enabled us to identify the noise kernel [16–18] and adopt the proper statistical mechanical interpretation of noise in quantum field theory, the decoherence functional of Gell-Mann and Hartle and the consistent history formalism of Griffith and Omnes address the decoherence of histories and the emergence of quasiclassical domains. These three formalisms are shown (or demonstrated in specific models) to be closely related. (For the relation between CTP and IF, see refs. 18 and 19, for that between IF and DF, see refs. 20, 21, and 23.) They constitute the formal basis for establishing a new regime between semiclassical and quantum physics named the stochastic (semiclassical) regime.

Thus, viewing semiclassical gravity as an open system enabled us to link up with inquiries of a fundamental nature such as the relation of classical, stochastic, and quantum and tap into the conceptual and technical resources in this endeavor. It opened up a new horizon where some of the basic issues of quantum mechanics such as decoherence and the emergence of the classical world (classical spacetime) can be addressed in statistical mechanical theoretical terms; and the formal tools of quantum field theory such as the effective action method can be used to quantify statistical mechanical notions and depict processes such as dissipation and noise (from activities of the quantum vacuum of matter fields). With these ideas and methods at work, the stage was set around 1993 for probing into a deeper level of structure of gravity beyond the semiclassical theory, which we call stochastic semiclassical gravity. (For a summary of work in this first stage 1989–1993, see, e.g., refs. 24 and 25.)

1.3. Einstein–Langevin Equation

The next 3 years saw the developement of such a theory centering on the quantification of noises associated with quantum field processes [16] and the discovery of a new equation in this regime known as the Einstein–Langevin equation [18, 25–27] which relates the dissipative dynamics of spacetime and the fluctuations in the quantum matter fields. It has the form of a semiclassical Einstein equation (which contains a dissipative term from the dissipation kernel in the influence action), but with an additional stochastic source term (from the noise kernel in the influence action).

Let me illustrate this theory with a brief sketch of the example of a conformally coupled scalar field in a weakly perturbed (anisotropic or inhomogeneous) spatially flat Friedmann–Lemaitre–Robertson–Walker (FLRW) universe with metric $g^{RW}_{\mu\nu}$ plus small perturbations $h_{\mu\nu}$,

$$g_{\mu\nu}(x) = g^{RW}_{\mu\nu} + h_{\mu\nu} \equiv a(\eta)^2 \tilde{g}_{\mu\nu} \tag{1}$$

Here η is the conformal time related to the cosmic time t by $dt = a(\eta)\, d\eta$. In this form the metric is conformally related [via conformal factor $a(\eta)$] to the Minkowski metric $\eta_{\mu\nu}$ and its perturbations $\tilde{h}_{\mu\nu}(x)$:

$$\tilde{g}_{\mu\nu} = \eta_{\mu\nu} + \tilde{h}_{\mu\nu}(x) \tag{2}$$

The perturbations $h_{\mu\nu}$ can be homogeneous (the case $= \delta a^2$ was treated by Calzetta and Hu [18]), or anisotropic (as in a Bianchi type I case treated by Hu and Sinha [25]), or inhomogeneous (treated by Campos, Martin, and Verdaguer [27, 23]). Here we follow the latter work.

The classical action for a free massless real scalar field $\Phi(x)$ is given by

$$S_f[g_{\mu\nu}, \Phi] = -\frac{1}{2}\int d^n x \ \sqrt{-g}[g^{\mu\nu}\,\partial_\mu\Phi\,\partial_\nu\Phi + \xi(n)R\Phi^2] \tag{3}$$

where R is the Ricci curvature scalar for the metric $g_{\mu\nu}$ and $\xi(n) = (n-2)/[4(n-1)]$, n being the spacetime dimension, is the coupling of the field to the spacetime, with $\xi(4) = 0$ and $1/6$ corresponding to minimal and conformal couplings, respectively. We consider a massless conformally coupled scalar field here. Define a conformally related field $\Phi(x) \equiv a(\eta)^{(n/2-1)}\Phi(x)$; the action S_f (after one integration by parts)

$$S_f[\tilde{g}_{\mu\nu}, \Phi] = -\frac{1}{2}\int d^n x \ \sqrt{-\tilde{g}}[\tilde{g}^{\mu\nu}\,\partial_\mu\Phi\,\partial_\nu\Phi + \xi(n)\tilde{R}\Phi^2] \tag{4}$$

takes the form of an action for a free massless conformally coupled real scalar field $\Phi(x)$ in a spacetime with metric $\tilde{g}_{\mu\nu}$. In this case it is a nearly flat spacetime. As the physical field $\Phi(x)$ is related to the field $\Phi(x)$ by a power of the conformal factor, a positive-frequency mode of the field $\Phi(x)$ in flat spacetime will correspond to a positive-frequency mode in the conformally related space. One can thus establish a quantum field theory in the conformally related space by use of the conformal vacuum [1]. Quantum effects such as particle creation arises from the breaking of conformal flatness of the spacetime produced by the perturbations $h_{\mu\nu}(x)$.

1.3.1. Semiclassical Einstein Equation

The Einstein equation for classical gravity is

$$G_{\mu\nu}[g] + \Lambda g_{\mu\nu} = 8\pi G T^{c}_{\mu\nu} \tag{5}$$

where G is the Newton constant, Λ is the cosmological constant, $g_{\mu\nu}$ is the spacetime metric, $G_{\mu\nu}$ is the Einstein tensor, and $T^{c}_{\mu\nu}$ is the energy-momentum tensor of classical matter or fields. One now adds the quantum field as a source, and gets the semiclassical Einstein equation (SCE)

$$G_{\mu\nu}[g] + \Lambda g_{\mu\nu} = 8\pi G (T^{c}_{\mu\nu} + T^{q}_{\mu\nu}) \tag{6}$$

where $T^{q}_{\mu\nu} \equiv \langle T_{\mu\nu}\rangle_q$ is the expectation value of the stress tensor operator in some quantum state of the matter field Φ. In general there are ultraviolet divergences in $\langle T^{\mu\nu}\rangle_q$. To remove or cure them one introduces regularization or renormalization procedures by adding counterterms or absorbing them into the cosmological constant, the Newton constant, and the coupling constants of the curvature-squared terms corresponding to the quartic, quadratic, and logarithmic divergences [1]. As a result the renormalized SCE equation takes the form

$$(G_{\mu\nu}[g] + \Lambda g_{\mu\nu}) - l_P^2\,(\alpha A_{\mu\nu} + \beta B_{\mu\nu})\,[g] = 8\pi G \langle \hat{T}^{R}_{\mu\nu}\rangle\,[g] \tag{7}$$

where G, Λ, α, and β are now renormalized coupling constants and $l_P \equiv \sqrt{16\pi G}$ is the Planck length. $A_{\mu\nu}$ and $B_{\mu\nu}$ are divergenceless local curvature tensors defined by

$$\begin{aligned} A^{\mu\nu}(x) &\equiv \frac{1}{\sqrt{-g}} \frac{\delta}{\delta g_{\mu\nu}} \int d^4x\ \sqrt{-g}\, C_{\alpha\beta\rho\sigma}C^{\alpha\beta\rho\sigma} \\ &= \frac{1}{2} g^{\mu\nu}\, C_{\alpha\beta\rho\sigma}C^{\alpha\beta\rho\sigma} - 2R^{\mu\alpha\beta\rho}R^{\nu}_{\alpha\beta\rho} + 4R^{\mu\alpha}R^{\nu}_{\alpha} \\ &\quad - \frac{2}{3} RR^{\mu\nu} - 2\Box_g R^{\mu\nu} + \frac{2}{3} R^{;\mu\nu} + \frac{1}{3} g^{\mu\nu}\Box_g R \end{aligned} \tag{8}$$

where $C_{\alpha\beta\rho\sigma}$ is the Weyl tensor, and

$$\begin{aligned} B^{\mu\nu}(x) &\equiv \frac{1}{\sqrt{-g}} \frac{\delta}{\delta g_{\mu\nu}} \int d^4x\ \sqrt{-g} R^2 \\ &= \frac{1}{2} g^{\mu\nu} R^2 - 2RR^{\mu\nu} + 2R^{;\mu\nu} - 2g^{\mu\nu}\Box_g R \end{aligned} \tag{9}$$

where R is the Ricci scalar. The divergence-free tensor $\langle \hat{T}^R_{\mu\nu} \rangle [g]$ is the expectation value in some quantum state of the renormalized stress tensor operator $\hat{T}^R_{\mu\nu}[g]$ of the field Φ.

For a massless conformally coupled scalar field in the metric (1) above, $\langle T^{\mu\nu} \rangle_q$ has the form (the subscripts 0, 1 in parentheses denote, respectively, zeroth and first order in $h_{\mu\nu}$) [27]

$$\langle T^{\mu\nu}_{(0)} \rangle_q = \lambda \left[H^{\mu\nu}_{(0)} - \frac{1}{6} B^{\mu\nu}_{(0)} \right]$$

$$\langle T^{\mu\nu}_{(1)} \rangle_q = \lambda \left[(H^{\mu\nu}_{(1)} - 2R^{(0)}_{\alpha\beta} C^{\mu\alpha\nu\beta}_{(1)}) - \frac{1}{6} B^{\mu\nu}_{(1)} \right.$$
$$\left. + 3a^{-3} \left(-4(C^{\mu\alpha\nu\beta}_{(1)} lna)_{,\alpha\beta} + \int d^4y \, A^{\mu\nu}_{(1)}(y) K(x-y; \bar{\mu}) \right) \right] \tag{10}$$

where the constant $\lambda = 1/2880\pi^2$ characterizes one-loop quantum correction terms (which include the trace anomaly and particle creation processes) and $\bar{\mu}$ is a renormalization parameter. Here $H^{\mu\nu}(x)$ arises from the counterterms in the renormalization of the energy-mementum tensor (see, e.g., ref. 28) and is related to $A^{\mu\nu}$, $B^{\mu\nu}$ above:

$$H^{\mu\nu}(x) \equiv -R^{\mu\alpha} R^{\nu}_{\alpha} + \tfrac{2}{3} R R^{\mu\nu} + \tfrac{1}{2} g^{\mu\nu} R_{\alpha\beta} R^{\alpha\beta} - \tfrac{1}{4} g^{\mu\nu} R^2 \tag{11}$$

I call attention to the existence of a dissipation term (kernel K) above describing the backreaction of particle creation on the background spacetime dynamics [3, 22]. All terms in the semiclassical Einstein equation originating from renormalization and backreaction, including the dissipative kernel, are familiar from model calculations done in the 1970s and 1980s (see, e.g., refs. 2). One needs the CTP effective action [4] to derive the correct SCE which is real and causal [3, 22].

1.3.2. Stochastic Semiclassical Einstein Equation

The stochastic semiclassical Einstein or Einstein–Langevin equation (ELE) [18, 26, 25] differs from the semiclassical Einstein equation (SCE) by the presence of a stochastic term measuring the fluctuations of quantum sources (arising from the difference of particles created in neighboring histories [18]) which is intrinsically linked to the dissipation term in the dynamics of spacetime. Two points are noteworthy: (a) The fluctuations and dissipation kernels (decipherable from the influence action) obey a fluctuation-dissipation relation which embodies the backreaction effects of quantum fields on classical spacetime. (b) The stochastic source term engenders metric fluctuations.

The semiclassical Einstein equation depicts a mean-field theory which one can retrieve from the Einstein–Langevin equation by taking a statistical average with respect to the noise distribution. We used the influence functional formalism to extract these new information [25, 27]. The stochastic semiclassical Einstein equation, or Einstein–Langevin equation, takes the form

$$G_{\mu\nu}[g] + \Lambda g_{\mu\nu} = 8\pi G(T^{c}_{\mu\nu} + T^{qs}_{\mu\nu})$$

$$T^{qs}_{\mu\nu} \equiv \langle T_{\mu\nu}\rangle_q + T^{s}_{\mu\nu} \tag{12}$$

The new term $T^{s}_{\mu\nu} = 2\tau_{\mu\nu}$, which is of classical stochastic nature, measures the fluctuations of the energy-momentum tensor of the quantum field. Define

$$\hat{t}_{\mu\nu}(x) \equiv \hat{T}_{\mu\nu}(x) - \langle \hat{T}_{\mu\nu}(x)\rangle \hat{I} \tag{13}$$

(Such a tensor is computed in the backgound metric, not the perturbed metric.) It is related to the noise kernel $N_{\mu\nu\rho\sigma}$ bitensor by

$$4N_{\mu\nu\rho\sigma}(x, y) \equiv \tfrac{1}{2}\langle\{\hat{t}_{\mu\nu}(x), \hat{t}_{\rho\sigma}(y)\}\rangle \tag{14}$$

where $\{\,\}$ means taking the symmetric product. The noise kernel appears in the real part of the influence action.

The noise kernel is free of ultraviolet divergence, as one can see from its definition and the fact that the ultraviolet behavior of $\hat{T}_{\mu\nu}$ and $\langle\hat{T}_{\mu\nu}\rangle$ is the same; thus one can replace $\hat{T}_{\mu\nu}$ by $\hat{T}^{R}_{\mu\nu}$ in these equations. The noise kernel defines a real, classical Gaussian stochastic symmetric tensor field $\tau_{\mu\nu}$ which is characterized to lowest order by the following correlators:

$$\langle \tau_{\mu\nu}(x)\rangle_\tau = 0, \qquad \langle \tau_{\mu\nu}(x)\tau_{\rho\sigma}(y)\rangle_\tau = N_{\mu\nu\rho\sigma}(x, y) \tag{15}$$

where $\langle\rangle_\tau$ means taking a statistical average (for simplicity no higher order correlations are assumed). Since $\hat{T}^{R}_{\mu\nu}$ is self-adjoint, one can see that $N_{\mu\nu\rho\sigma}$ is symmeric, real, positive, and semidefinite. Furthermore, as a consequence of (14) and the consevation law $\nabla^{\mu}\hat{T}^{R}_{\mu\nu} = 0$, this stochastic tensor is divergenceless in the sense that $\nabla^{\mu}\tau_{\mu\nu} = 0$ is a deterministic zero field. Also $g^{\mu\nu}\tau_{\mu\nu}(x) = 0$, signifying that there is no stochastic correction to the trace anomaly (if $T_{\mu\nu}$ is traceless). Here all covariant derivatives are taken with respect to the background metric $g_{\mu\nu}$, which is a solution of the semiclassical equations. Taking the statistical average of (12), as a consequence of the noise correlation relation (15),

$$\langle T^{qs}_{\mu\nu}\rangle_\tau = \langle T_{\mu\nu}\rangle_q \tag{16}$$

we recover the semiclassical Einstein equation (6).

Now for a spacetime with background metric $g_{\mu\nu}$ and weak gravitational perturbation $h_{\mu\nu}$ the EL equation to linear order in $h_{\mu\nu}$ has the form

$$(G_{\mu\nu}[g+h] + \Lambda(g_{\mu\nu} + h_{\mu\nu})) - 2l_P^2(\alpha A_{\mu\nu} + \beta B_{\mu\nu})[g+h]$$
$$= 8\pi G(\langle \hat{T}^R_{\mu\nu}\rangle[g+h] + 2\tau_{\mu\nu}) \tag{17}$$

The symmetry and divergenceless of the stochastic tensor in the background metric guarantee the consistency of this semiclassical Einstein–Langevin equation. This equation gives the first-order correction to semiclassical gravity in the sense that it incorporates the correlation of $T_{\mu\nu}$. The distinct feature is that it predicts the existence of a stochastic component in the metric $h^s_{\mu\nu}$ which we call metric *fluctuations*. It is induced by the quantum stress tensor fluctuations. Since the stress tensor fluctuations are defined on the background metric $g_{\mu\nu}$, the stochastic field $\tau_{\mu\nu}$ does not depend on the metric *perturbations* $h^c_{\mu\nu}$. Therefore Eq. (17) is a linear stochastic equation for $h_{\mu\nu}$ with an inhomogeneous term $\tau_{\mu\nu}$; its solution can be formally written as the functional $h_{\mu\nu}[\tau]$. Taking the statistical average of Eq. (17), one sees that the metric $g_{\mu\nu} + \langle h_{\mu\nu}\rangle_\tau$ must be a solution of the semiclassical Einstein equation linearized around $g_{\mu\nu}$. By the gauge invariance of Eq. (17) it is clear that if $h_{\mu\nu}$ is a solution of this equation, $h'_{\mu\nu} = h_{\mu\nu} + \nabla_\mu\zeta_\nu + \nabla_\nu\zeta_\mu$, where $\zeta_\mu(x)$ is a Gaussian stochastic field on the background spacetime, is a physically equivalent solution.

For the example of a perturbed spatially flat FLRW universe with a quantum scalar field Φ the tensor $\tau_{\mu\nu}(x)$ is given by [27]

$$\tau_{\mu\nu}(x) = -2\partial_\alpha\,\partial_\beta\xi^{\mu\alpha\nu\beta}(x) \tag{18}$$

which is seen to be symmetric and traceless, i.e., $\tau_{\mu\nu}(x) = \tau_{\nu\mu}(x)$ and $\tau^\mu_\mu(x) = 0$. The stochastic correction to the stress tensor has vanishing divergence with respect to the background metric.

In this problem the tensor $\xi_{\mu\nu\alpha\beta}(x)$ has the symmetries of the Weyl tensor, i.e., it has the symmetries of the Riemann tensor and vanishing trace in all its indices. It is characterized completely by the noise kernel $N(x-y)$ (the probability distribution for the noise is Gaussian) [25, 27]

$$\langle \xi_{\mu\nu\alpha\beta}(x)\rangle_\xi = 0$$
$$\langle \xi_{\mu\nu\alpha\beta}(x)\xi_{\rho\sigma\lambda\theta}(y)\rangle_\xi = T_{\mu\nu\alpha\beta\rho\sigma\lambda\theta}N(x-y) \tag{19}$$

Here $T_{\mu\nu\alpha\beta\rho\sigma\lambda\theta}$ is the product of four metric tensors (in such a combination that the right-hand side of the equation satisfies the Weyl symmetries of the two stochastic fields on the left-hand side). Its explicit form is given in ref. 27.

As mentioned above, the new source term $2\tau_{\mu\nu}$ will produce a stochastic contribution $h^s_{\mu\nu}$ to the spacetime metric, i.e., $h_{\mu\nu} = h^c_{\mu\nu} + h^s_{\mu\nu}$. Considering a flat background spacetime [setting $a = 1$ in (1) and dropping the tilde on $h_{\mu\nu}$ for simplicity], one obtains, by adopting the harmonic gauge condition

$(h^s_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h^s)^{,\nu} = 0$, a linear equation for the metric fluctuations (off Minkowski spacetime here) $h^s_{\mu\nu}$

$$\Box h^s_{\mu\nu} = 16\pi G T^s_{\mu\nu}$$
$$T^s_{\mu\nu} = 2\tau_{\mu\nu} = -4\partial_\alpha\,\partial_\beta\,\xi^{\mu\alpha\nu\beta} \tag{20}$$

The computation of the noise correlations and the solution of these equations have been given by Campos and Verdaguer [27]. Calzetta *et al.* have solved the Einstein–Langevin equation for a cosmological problem with both noise and dissipation [29].[2] Recently Martin and Verdaguer [23] have revisited this problem. They solved the stochastic semiclassical Einstein equation (17) around the Minkowski spacetime $\eta_{\mu\nu}$ for a massless, conformally coupled scalar field in its vacuum state $|0\rangle$. In this case $\langle 0|T^R_{\mu\nu}[\eta]|0\rangle = 0$ and if we take $\Lambda = 0$, the Minkowski metric is a trivial solution of the semiclassical Einstein equation (7). Since the vacuum state is not an eigenstate of $\hat{T}^R_{\mu\nu}[\eta]$, fluctuations of the stress tensor are present. Equation (17) reduces in this case to the linearized SCE equations derived by Horowitz [30] for studying the semiclassical stability of flat spacetime, but with a new inhomogeneous source term $\tau_{\mu\nu}$. Martin and Verdaguer evaluated the two-point correlation function of the linearized Einstein tensor and found that for spacelike-separated points $\underline{x}$ and $\underline{x}'$ it goes like

$$\frac{1}{l_P^2}\frac{1}{|\underline{x} - \underline{x}'|^2}\exp\left(-\frac{|\underline{x} - \underline{x}'|}{l_P}\right) \tag{21}$$

The above result shows that the quantum field fluctuations induce metric fluctuations with a correlation length l_P. The appearence of Planck length here is not surprising since for a massless scalar field coupled to gravity there is no other length scale in the problem. It is noteworthy that this result is not analytic in l_P and thus it could not have been obtained by a perturbative expansion in the Planck length. Of course, this semiclassical result is expected to break down at Planck scale, and quantum fluctuations of the metric beyond that induced by linear perturbations (gravitons can be treated as a quantum field source as each is identical to two components of massless, minimally coupled scalar field) would become important.

For other recent developments, I would like to mention a derivation of the EL equation from renormalization group considerations by Lombardo

[2] A comment from E. Verdaguer: In Eq. (20), Campos and Verdaguer [27] neglected the dissipation term, or more precisely, the expectation value of the stress tensor to linear order in $h_{\mu\nu}$ which is of the same order as $T^s_{\mu\nu}$ is stochastic. For this reason the solution they found was only formal; it is divergent in fact. One can get a finite result if one starts the perturbation at some initial time zero (in the paper they start at $t = -\infty$). This is similar to having a particle in a bath with no dissipation for a very long time; the fluctuations will take it very far. In ref. 29 the calculation was only for the homogeneous conformal mode.

and Mazzitelli [31], and the application of the CTP-IF formalism to the study of backreaction of Hawking radiation in 2D dilatonic black hole spacetimes by Lombardo *et al.* [32].

1.4. Stochastic Gravity in Relation to Semiclassical and Quantum Gravity

Stochastic gravity is a regime intermediate between semiclassical and quantum gravity. It is perhaps instructive to examine the distinction among these three theories.

We use the example above for gravitational perturbations $h_{\mu\nu}$ in a FLRW universe with background metric $g_{\mu\nu}$ driven by the expectation value of the energy-momentum tensor of a scalar field Φ, as well as its fluctuations $\hat{t}_{\mu\nu}(x)$. Let us compare the stochastic with the semiclassical and quantum equations of motion for the metric perturbation field h (we will use schematic notations for simplicity). The semiclassical equation is given by

$$\Box h = \langle \hat{T} \rangle \tag{22}$$

where $\langle\rangle$ denotes taking the quantum average (e.g., the vacuum expectation value) of the operator enclosed. Its solution can be written in the form

$$h = \int G\langle \hat{T} \rangle, \qquad h_1 h_2 = \int\int G_1 G_2 \langle \hat{T} \rangle \langle \hat{T} \rangle \tag{23}$$

The quantum (Heisenberg) equation

$$\Box \hat{h} = \hat{T} \tag{24}$$

has solutions

$$\hat{h} = \int G\hat{T}, \qquad \langle \hat{h}_1, \hat{h}_2 \rangle = \int\int G_1 G_2 \langle \hat{T}\hat{T} \rangle_{\hat{h},\hat{\phi}} \tag{25}$$

where the average is taken with respect to the quantum fluctuations of both the gravitational and the matter fields. Now for the stochastic equation, we have

$$\Box h = \langle \hat{T} \rangle + \tau \tag{26}$$

with solutions[3]

[3] In this schematic form we have not displayed the homogeneous solution carrying the information of the (maybe random) initial condition. This solution will exist in general, and may even be dominant if dissipation is weak. When both the uncertainty in initial conditions and the stochastic noise are taken into account, the Einstein–Langevin formalism reproduces the exact graviton two-point function in the linearized approximation. Of course, it fails to reproduce the expectation value of observables which could not be written in terms of graviton occupation numbers, and in this sense it falls short of full quantum gravity. I thank E. Calzetta for this comment.

$$h = \int G\langle \hat{T} \rangle + \int G\tau, \qquad h_1 h_2 = \int\int G_1 G_2 [\langle \hat{T} \rangle \langle \hat{T} \rangle + (\langle \hat{T} \rangle \tau + \tau \langle \hat{T} \rangle) + \tau\tau] \tag{27}$$

We now take the noise average $\langle\rangle_\xi$. Recall that the noise is defined in terms of the stochastic sources τ as

$$\langle \tau \rangle_\xi = 0, \qquad \langle \tau_1 \tau_2 \rangle_\xi \equiv \langle \hat{T}_1 \hat{T}_2 \rangle - \langle \hat{T}_1 \rangle \langle \hat{T}_2 \rangle \tag{28}$$

we get

$$\langle h_1 h_2 \rangle_\xi = \int\int G_1 G_2 \langle \hat{T}\hat{T} \rangle_\phi \tag{29}$$

Note that the correlation of the energy-momentum tensor appears just as in the quantum case, but the average here is only over noise from quantum fluctuations of the matter field.

As seen above, while the semiclassical regime describes the effect of a quantum matter field only through its mean value (vacuum expectation value), the stochastic regime includes the fluctuations of quantum fields as reflected in the new stochastic term in the energy-momentum tensor. Thus stochastic gravity carries some information about the correlation of fields (and the related phase information) which is absent in semiclassical gravity. Here we have invoked the relation between fluctuations and correlation, a variant form of the fluctuation-dissipation relation. This feature pushes stochastic gravity closer than semiclassical gravity to quantum gravity in that the correlation in quantum field and geometry fully present in quantum gravity is partially retained in stochastic gravity, and the background geometry has a way to sense the correlation of the quantum fields through the noise term in the Einstein–Langevin equation, which shows up as metric fluctuations.

Thus ‘noise’ as used in this more precise language and context is not something one can arbitrarily assign or relegate, but has taken on a wider meaning in that it embodies the contributions of the higher correlation functions in the quantum field. Only the lowest order is being displayed in what has been done so far, in terms of the two-point function of the energy-momentum tensor (or the four-point function of fields). Although the Feynman–Vernon way can only accommodate Gaussian noise of the matter fields and takes a simple form for linear coupling to the background spacetime, the notion of noise can be made more general and precise. (For an example of more complex noise associated with more involved backreactions arising from strong or nonlocal coupling, see Johnson and Hu [33].) Progress is made now on how to characterize the higher order correlation functions of an interacting field systematically from the Schwinger–Dyson equations in terms of what Calzetta and I called ‘correlation noise’ [34, 35], after the BBGKY hierarchy. This will be discussed in a later section.

Notice also that the difference between stochastic gravity and quantum gravity is that in the former only the fluctuations and correlations of matter fields are accounted for, while the full quantum theory should also include the fluctuations and correlations of the quantum gravitational field. We will focus on this difference and discuss how closely one could probe into the full theory with stochastic equations later.

The aim of this paper is to deliberate on the meanings of this new regime, the significance of quantum noise and metric fluctuations in affecting Planck-scale processes, and how correlation bears to reveal a deeper level of spacetime structure short of knowing the full theory of quantum gravity. I will also describe some ongoing research in this program and make suggestions for further investigations.

2. METRIC FLUCTUATIONS FROM BACKREACTION OF QUANTUM FIELDS

By construction this new framework is suitable for investigation into metric fluctuations and backreaction effects. So far it has been applied [25, 27, 29] to study quantum effects in cosmological spacetimes. Work on black hole spacetimes has just begun [36–39]. Parallel to this is the interesting application to noise-induced phase transitions by Calzetta and Verdaguer [40]. I will also mention other directions, including applications in thermal field theories (hot flat space) [41].

2.1. Metric Fluctuations in Semiclassical Gravity

Metric fluctuation and its more colorful generalization called spacetime foam have been a subject of intermittent speculations since Wheeler introduced it in the early 1960s to address ‘the issue of the final state’ in general relativity [42]. We will have more to say about this generalization from the viewpoint of stochastic gravity later. Here it is sufficient to point out that the correlation functions for the noise kernels obtained by Calzetta and Hu [18], Hu and Matacz [26], Hu and Sinha [25], and Campos, Martin, and Verdaguer [27, 23] (see last section) give the first quantitative description of metric fluctuations as induced by quantum fields. To begin, it is perhaps useful to emphasize the difference in the meaning of ‘metric fluctuations’ used in our program, which includes backreaction from quantum fields, and that used by many others in the test field context, where one considers classical gravitational perturbations $h_{\mu\nu}$ from a fixed background geometry and their two-point functions $\langle h_{\mu\nu}(x)h_{\rho\sigma}(y)\rangle$ (averaged with respect to some vacuum, in a semiclassical sense).[4] It is useful as a measure of the fluctuations in the gravitational field at particular regions of spacetime. Ford and coworkers have explored this aspect in great detail [43] (They call this kind of fluctuation 'active' and the kind we discuss here 'passive'—I would prefer to call them 'spontaneous' and 'induced'.) However, when backreaction is included, as is necessary at the Planck scale, with the background spacetime metric and the quantum fields present evolving together consistently, the graviton two-point function calculated with respect to a fixed background (as in the case of 'active' fluctuations) rapidly loses its relevance.

In contrast, metric fluctuations $h^s_{\mu\nu}$ here [18, 25–27] are defined for semiclassical gravity in the backreaction context. They are classical stochastic quantities arising from the flucutations in the quantum fields present and are important only at the Planck scale. We see that they are derived from the noise kernel, which, if the quantum field is the graviton, involves graviton four-point functions. It is this quantity which enters into the fluctuation-dissipation relation—not the usual graviton two-point function—which encapsulates the semiclassical backreaction.

An immediate application of metric fluctuations is on the stability of semiclassical spacetimes (solutions to the semiclassical Einstein equations) against stochastic sources from particle creations, and the validity of semiclassical gravity. The determining factor is in the noise kernel, which is related to the fluctuations of the energy-momentum tensor. Kuo and Ford [44] calculated the fluctuations in the Casimir energy density for flat space and found it to be comparable to the mean. Phillips and Hu [45] confirmed their result using a covariant generalized zeta function method.

For quantum fields in a curved spacetime with an Euclidean section, Phillips and Hu [45] derived a general expression for the stress-energy tensor two-point function in terms of the effective action. The renormalized two-point function is given in terms of the second variation of the Mellin transform of the trace of the heat kernel for the quantum fields. For systems in which a spectral decomposition of the wave operator is possible, one can derive an exact expression for this two-point function. As a measure of the magnitude of fluctuations dimensionless expression for the ratio between the variance of each component of the stress-energy tensor compared to the mean is used [44]:

$$\Delta_{abcd}(x) = \left| \frac{\langle T_{ab}(x)T_{cd}(x)\rangle_{\rm ren} - \langle T_{ab}(x)\rangle_{\rm ren}\,\langle T_{cd}(x)\rangle_{\rm ren}}{\langle T_{ab}(x)T_{cd}(x)\rangle_{\rm ren}} \right| \qquad (30)$$

[4] The two-point functions of gravitons are not stochastic variables and so in a stricter sense they should not be called metric 'fluctuations.' To avoid confusion we may at times call our quantities $h^s_{\mu\nu}$ induced metric fluctuations.

From inspection, $0 \leq \Delta_{abcd} \leq 1$. Only for $\Delta \ll 1$ can the fluctuations be viewed as small. On the other hand, $\Delta \sim 1$ indicates that the fluctuations can be large compared to the mean value. Phillilps and Hu studied two cases in detail with this method: d-dimensional flat space with one periodic dimension ($R^d \times S^1$) with a minimally coupled massless scalar field and the Einstein universe (S^3) with a conformally coupled massless scalar field. The results for the energy density are (τ denotes Euclildean time):

$$\Delta_{\tau\tau\tau\tau} (R^d \times S^1) = \frac{(d+1)(d+2)}{(d+1)(d+2)+2}, \qquad \Delta_{\tau\tau\tau\tau}(S^3) = \frac{111}{112} \sim .99 \tag{31}$$

The large variance signifies the importance of quantum fluctuations and may indicate the breakdown of semiclassical gravity at sub-Planckian scales.

2.2. Black Hole Fluctuations and Backreaction

Work in progress now focuses on fluctuations of the energy density of quantum fields in early universe and black hole spacetimes. These results will have direct bearing on structure formation from quantum fluctuations in the early universe (see, e.g., ref. 46 and references therein) and stability of black holes against Hawking radiation and the related entropy and information loss issues. Here, as before, the central task is the computation of the noise kernel, or the fluctuations of the energy-momentum tensor. One can use the zeta function method (as in Phillips and Hu [45]) for treating the second variation of the effective action, or more explicitly, the covariant point splitting method [47]. The main difficulty for black hole spacetimes, as is already present in the calculation of the regularized energy-momentum tensor for spherically symmetric spacetimes [48], lies in the radial functions. For optical metrics one can use the Gaussian approximation for the propagators as was done by Page [49], who obtained an expression for the energy density of quantum scalar fields which was shown to be good to an unexpectedly high accuracy. Phillips [37] obtained results for the fluctuations of the energy density of a scalar field in a general optical metric and is in progress for the Schwarzschild metric at the horizon. Earlier, Ford [50] showed that (spontaneous) black hole horizon fluctuations—the graviton two-point function—are much smaller than Planck dimensions for black holes whose mass exceeds the Planck mass. From our result and Ford's, one sees that, contrary to some recent claims [51], the semiclassical derivation of Hawking radiance should remain valid for black holes larger than the Planck mass and there is no drastic effect near the horizon arising from metric fluctuations. Other recent work on black hole horizon (spontaneous) fluctuations includes Sorkin [52] and Barrabès *et al.* [53].

The cosmological backreaction problem saw two stages of development as represented by the use of the in-out cum in-in effective action (e.g., ref. 3) followed by the influence action (e.g., ref. 25) for extracting first the mean value and then the fluctuations of the energy-momentum tensor, which physically corresponds to the study of dissipation and fluctuations of the spacetime. Likewise, the black hole backreaction problem also progressed in two stages. The first stage started in the early 1980s with the work of Candelas, Howard, Page, Frolov, Jensen, McLaughlin, Ottiwell, Hiscock, Anderson, and others (see ref. 48 and references therein) in the calculation of the regularized energy-momentum tensor for quantum fields in black hole spacetimes. The second stage has just begun. It focuses on calculating the fluctuations of the energy-momentum tensor as described above, and with it the backreaction on the black hole spacetime configurations and dynamics. Ref. 39 gives a loose sketch of our program of investigation. We discussed the formulation of the problem, commenting on possible advantages and shortcomings of existing works, and introduced our own approach via stochastic semiclassical gravity. The goal is to derive and solve the Einstein–Langevin equation (or its physical equivalent, the fluctuation-dissipation relation) for a self-consistent description of metric fluctuations and the dissipative dynamics of a black hole with backreaction from its radiance. We divided the problem into two main classes, the quasistatic problem and the dynamic problem. The quasistatic problem is characterized by a black hole in quasiequilibrium with its Hawking radiation (enclosed in a box to ensure relative stability). One important early work on backreaction of this kind is by York [55], while the most thorough work to date is Anderson *et al.* [48]. Backreaction for dynamical (collapsing) black holes is much more difficult to treat than static ones, and there are fewer viable attempts. For situations with black hole masses much greater than the Planck mass, one important early work which captures the overall features of dynamical backreaction is that by Bardeen [56] and its further elaboration by Massar [57]. (See ref. 39 for more details.)

2.3. Fluctuation-Dissipation Relation for Black Holes

Candelas and Sciama [58] were the first to suggest that the black hole radiance problem can be understood as a quantum dissipative system. For a static black hole in equilibrium with its Hawking radiation, Mottola [95] used the formal equivalence to a thermal field problem to show that in the Hartle–Hawking state a fluctuation-dissipation relation (FDR) exists between the expectation values of the commutator and anticommutator of the energy-momentum tensor of the scalar field, a form familiar in linear response theory [14]. In Ref. 39 we showed how both of these proposals are flawed. We showed why for a *bona fide* backreaction study of thermal radiance on a quasistatic black hole, one should consider *ab initio* states more general than the Hartle–Hawking state. To obtain a causal fluctuation-dissipation relation (FDR) one needs to use the in-in (or Schwinger–Keldysh) formalism applied to a class of quasistatic metrics (generalization of York [55]) and calculate the fluctuations of the energy-momentum tensor for the noise kernel. So far we have completed such a calculation [41] only for thermal fields in a weak gravitational field which depicts the far-field limit of a Schwarzschild black hole spacetime [60]. For the noise kernel of quantum fields near a Schwarzschild horizon Phillips [37] obtained a finite expression using the Gaussian approximation for the Green function. The accuracy of this approximation worsens near the horizon and a more reliable calculation would require the inclusion of higher order terms in the Schwinger–DeWitt expansion (the a_1, a_2 coefficients). In the following I outline the recent result of Campos and Hu [41] for thermal fields in a weak gravitational background, which can be viewed as the far-field limit of black hole spacetimes.

2.4. Thermal Fields in Black Hole Spacetimes

The behavior of a relativistic quantum field at finite temperature in a weak gravitational field has been studied before by a number of groups [61–63] for scalar and Abelian gauge fields. In these works the thermal graviton polarization tensor and the effective action have been calculated and applied to the study of the stability of hot flat and curved spaces and the "dynamics" of cosmological perturbations. To describe screening effects and stability of thermal (linearized) quantum gravity, one needs only the real part of the polarization tensor, but for damping effects, the imaginary part is essential. The gravitational polarization tensor obtained from the thermal graviton self-energy represents only a part (the thermal correction to the vacuum polarization) of the finite-temperature quantum stress tensor. There are in general also contributions from particle creation (from vacuum fluctuations at zero and finite temperatures). These processes engender dissipation in the dynamics of the gravitational field and their fluctuations appear as noise in the thermal field. We have found such a relation between these two processes, which embodies the backreaction self-consistently.

Our calculation of the quantum corrections of the scalar field to the thermal graviton polarization tensor was carried out by means of the Feynman–Vernon [8] influence functional (IF). It yields results identical to that obtained before by means of linear response theory (LRT) [62, 63]. From the IF one can obtain the noise and dissipation kernels explicitly which satisfy a fluctuation-dissipation relation (FDR) [13] at all temperatures.

We consider a free massless scalar field Φ arbitrarily coupled to a gravitational field $g_{\mu\nu}$ with classical action (3). In the weak-field limit we consider a small perturbation $h_{\mu\nu}$ from flat spacetime $\eta_{\mu\nu}$ in the form $g_{\mu\nu}(x) = \eta_{\mu\nu} + h_{\mu\nu}(x)$ with signature $(-, +, \cdots, +)$ for the Minkowski metric. The CTP effective action at finite temperature $T = 1/\beta$ for a free quantum scalar field in this gravitational background is given by

$$\Gamma^{\beta}_{CTP}[h^{\pm}_{\mu\nu}] = S^{div}_{g}[h^{+}_{\mu\nu}] - S^{div}_{g}[h^{-}_{\mu\nu}] - \frac{i}{2}\,\mathrm{Tr}\{\ln \bar{G}^{\beta}_{ab}[h^{\pm}_{\mu\nu}]\} \tag{32}$$

where $a, b = \pm$ denote the forward and backward time paths respectively, and $\bar{G}^{\beta}_{ab}[h^{\pm}_{\mu\nu}]$ is the complete 2×2 matrix propagator with thermal boundary conditions for the differential operator $\Box + V^{(1)} + V^{(2)} + \cdots$, where $V^{(n)}$ contain terms of nth order in $h_{\mu\nu}$ from the expansion of the scalar curvature in S_f. Here S^{div}_{g} is the (divergent) gravitational action

$$\begin{aligned} S^{div}_{g}[g_{\mu\nu}] = {} & \frac{1}{l_P^2}\int d^n x\ \sqrt{-g}R(x) \\ & + \frac{\lambda\bar{\mu}^{n-4}}{4(n-4)}\int d^n x\ \sqrt{-g}\Bigg[3R_{\mu\nu\rho\sigma}(x)R^{\mu\nu\rho\sigma}(x) \\ & - \left(1 - 360\left(\xi - \frac{1}{6}\right)^2\right)R(x)R(x)\Bigg] \end{aligned} \tag{33}$$

The first term is the classical Einstein–Hilbert action and the second (divergent) term in four dimensions is the counterterm introduced to renormalize the effective action. As before, $l_P^2 = 16\pi G$, $\lambda = (2880\pi^2)^{-1}$, and $\bar{\mu}$ is an arbitrary mass scale. (It is noteworthy that the counterterms are independent of the temperature because the thermal contribution to the effective action does not contain additional divergences.)

We skip the details [41] and quote the results. The noise and dissipation kernels are expressed in terms of the propagators $\tilde{G}^{\beta}_{\pm\mp}$ (here tilde indicates the Fourier transform and the $+/-$ signs indicate the time branches in CTP), respectively, as

$$\begin{aligned} \mathrm{N}^{\mu\nu,\rho\sigma}(k) = {} & -\frac{1}{4}\int \frac{d^4 q}{(2\pi)^4}\,[\tilde{G}^{\beta}_{-+}(k+q)\tilde{G}^{\beta}_{+-}(q) \\ & + \tilde{G}^{\beta}_{+-}(k+q)\tilde{G}^{\beta}_{-+}(q)]\mathrm{T}^{\mu\nu,\rho\sigma}(q, k) \end{aligned} \tag{34}$$

$$\begin{aligned} \mathrm{D}^{\mu\nu,\rho\sigma}(k) = {} & \frac{i}{4}\int \frac{d^4 q}{(2\pi)^4}\,[\tilde{G}^{\beta}_{-+}(k+q)\tilde{G}^{\beta}_{+-}(q) \\ & - \tilde{G}^{\beta}_{+-}(k+q)\tilde{G}^{\beta}_{-+}(q)]\mathrm{T}^{\mu\nu,\rho\sigma}(q, k) \end{aligned} \tag{35}$$

It is easy to show that they are related by the thermal identity

$$N^{\mu\nu,\rho\sigma}(k) = i \coth\left(\frac{\beta k^o}{2}\right) D^{\mu\nu,\rho\sigma}(k) \tag{36}$$

In coordinate space we have the analogous expression

$$N^{\mu\nu,\rho\sigma}(x) = \int d^4x' \, K_{FD}(x - x') \, D^{\mu\nu,\rho\sigma}(x') \tag{37}$$

where the fluctuation-dissipation kernel $K_{FD}(x - x')$ is given by the integral

$$K_{FD}(x - x') = i \int \frac{d^4k}{(2\pi)^4} \, e^{ik\cdot(x-x')} \coth\left(\frac{\beta k^o}{2}\right) \tag{38}$$

Defining the variance of the energy-momentum tensor of the thermal field $\hat{t}^{\mu\nu}_{\beta}(x) \equiv \hat{T}^{\mu\nu}(x) - \langle \hat{T}^{\mu\nu}(x)\rangle_{\beta}\hat{I}$, one can show that

$$\langle\{\hat{t}^{\mu\nu}_{\beta}(x), \hat{t}^{\rho\sigma}_{\beta}(x')\}\rangle_{\beta} = 8N^{\mu\nu,\rho\sigma}(x - x') \tag{39}$$

$$\langle[\hat{t}^{\mu\nu}_{\beta}(x), \hat{t}^{\rho\sigma}_{\beta}(x')]\rangle_{\beta} = 8iD^{\mu\nu,\rho\sigma}(x - x') \tag{40}$$

From the CTP effective action one can also derive an Einstein–Langevin equation governing the evolution of the gravitational field under the dynamical influence of the thermal field, with a stochastic source term whose autocorrelation is given by the noise kernel. This is not so easily obtainable by the conventional methods such as LRT in thermal field theory.

3. SEMICLASSICAL GRAVITY AS MESOSCOPIC PHYSICS

In the above I have sketched some current activities in stochastic gravity. As we have seen, the main issue in the stochastic regime is that of noise and fluctuations. Let us now explore its implications. In particular, what can we say about quantum gravity now that this new theory is supposedly one step closer to it than semiclassical gravity? In order to answer this question, we need to examine where the stochastic situation is placed between the semiclassical and quantum situations insofar as the main physical issues are concerned. We also need to discuss some philosophical issues related to how we view the structure and origin of spacetime and reexamine the meaning of quantizing gravity. For these we need first to ponder the relation between quantum and classical as well as micro- and macrophysics.

On this issue I have proposed to view the low-energy theory (classical GR) as the hydrodynamic collective state of the substructures of spacetime. Only these basic constituents (most likely fermions), and not the collective variables, obey quantum mechanical rules. (One can quantize these variables, but they describe excitations of the collective modes such as phonons, plasmons etc., not the underlying basic consistituents such as atoms or electrons.) In this view, quantum gravity does not refer to a quantization of metric or connections (which describe the collective modes), but to the more basic, as yet unknown (strings?) constituents. To see the effects of this deeper structure with its coherence properties from the stochastic regime we suggest to rely on topological signatures, effective field theory, and the correlation hierarchy and its dynamics. We shall put aside topological considerations and only address in the next three sections three groups of issues—stochasticity, collectivity, and correlations—following the themes "semiclassical gravity as mesoscopic physics" [64], "general relativity as geometro-hydrodynamics" [65, 66], and "quantum microdynamics via correlation hierarchy" [21, 34, 35]. I will spend less space on stochasticity even though it is the central theme of this new regime of interest, because it has been discussed extensively in recent articles [18, 25–27, 67]. Rather I will expand on the other two issues and indicate productive avenues for further investigation.

In an essay written in 1994 [64] I proposed to examine some important issues in semiclassical gravity in the light of mesoscopic physics, issues such as the transition from quantum to classical spacetime via decoherence, crossover behavior at the Planck scale, tunneling and particle creation, growth of density contrast from vacuum fluctuations, or finite-size effect in curved spacetime phase transitions, and indicate some basic concerns of mesoscopic physics for condensed matter, atoms or nuclei, in the quantum/classical and the micro/macro interfaces, or the discrete/continuum and the stochastic/deterministic transitions. I pointed out that underlying these issues are three main factors: quantum coherence, fluctuations, and correlation. I discussed how a deeper understanding of these aspects of fields and spacetimes can help us address some basic problems, such as Planck-scale metric fluctuations, cosmological phase transition and structure formation, and the black hole entropy, end-state, and information paradox.

Mesoscopic physics deals with problems where the characteristic interaction scales or sample sizes are intermediate between the microscopic and the macroscopic. For experts they refer to a specific set of problems in condensed matter and atomic/optical physics (see, e.g., refs. 68). For the present discussion, I will adopt a more general definition, with 'meso' referring to the interface between macro and micro on the one hand and the interface between classical and quantum on the other.[5] These two aspects will often bring in the continuum/discrete and the deterministic/stochastic factors. I showed how issues concerning the micro/macro interface and the quantum to classical transition arise in quantum cosmology and semiclassical gravity in a way categorically similar to the new problems arising from condensed matter and atomic/optical physics (and, at a higher energy level, particle/nuclear physics, at the quark-gluon and nucleon interface). Many issues are related to the coherence and correlation properties of quantum systems, and involve stochastic notions, such as noise, fluctuations, dissipation, and diffusion in the treatment of transport, scattering, and propagation processes. The advantage of making such a comparison between these two apparently disjoint disciplines is twofold: The theory of mesoscopic processes, which can be tested in laboratories with the newly developed nanotechnology, can enrich our understanding of the basic issues common to these disciplines while being extended to the realm of general relativity and quantum gravity. The formal techniques developed and applied to problems in quantum field theory and spacetime geometry can be adopted to treat condensed matter and atomic/optical systems with more rigor, accuracy, and completeness. Many conceptual and technical challenges are posed by mescoscopic processes in both areas.

3.1. Mesoscopic Physics—Fundamental Issues at the Quantum/Classical and Micro/Macro Interfaces

Viewed in a more theoretical light, we can discern three aspects which underlie all mesoscopic processes in gravitation and in condensed matter physics. They are quantum coherence, fluctuations, and correlations. They are manifest in the quantum/classical and the micro/macro interfaces.

[5] Another meaning of mesoscopic can be defined with respect to structures and interactions. Instead of dwelling on these individual processes in their specific context, one can refer to the general category of problems which exist between two distinct levels of matter structure or interaction scales, such as between the molecular and atomic scales, the QED lepton and hadron scales, the nucleon and particle (quark—gluon) scales, the QCD and GUT (grand unification theory) scales (with or without deserts in between), and, of course, from the GUT to the QG (quantum gravity) scale, which is depicted by semiclassical gravity. The distinct levels of interaction are not arbitrarily picked; they obey theories of a 'fundamental' (QED, QCD) or derived (atomic, nuclear interaction) nature—even what we today view as fundamental interactions may just be collective states of a deeper structure. The mesoscales between them have common traits. They usually fall in the range where the approximations taken from either level (e.g., low-energy QCD versus perturbative hadron physics) fail, and new structure depicted by new collective variables and new language is called for. The new problems encountered in condensed matter and nuclear/particle physics fall under such a conceptual category, as do the problems of extending semiclassical gravity toward quantum gravity or projecting quantum gravity (e.g., superstring theory) onto low-energy particle physics (the standard model).

3.1.1. Fluctuations and Decoherence

Fluctuations and noise in the environment are responsible for decoherence in the system, which is a necessary condition for the quantum-to-classical transition [12, 11]. Classical description in terms of definite trajectories in phase space requires correlations between conjugate variables. Noise and fluctuations destroy this correlation. The observed classical reality as an emergent phenomenon from the quantum description has intrinsic stochastic behavior [15, 10].

3.1.2. Coherence and Dissipation

This is the counterpart to the above, as fluctuations and dissipation are balanced by the fluctuation-dissipation relation. The degree of coherence here refers to the phase information in a quantum system which can be corrupted by its interaction with an environment, resulting in a stochastic classical dynamics for the system. Coherence in quantum systems is altered by dissipative effects, as occurs in macroscopic quantum phenomena [69], e.g., in tunneling with dissipation at finite temperature.

3.1.3. Correlation and Collectivity

A useful signifier of the statistical properties of a system is its correlation functions—the BBGKY hierarchy in classical physics, or the Schwinger–Dyson equations in quantum field theory. It can be used to measure the degree of coherence in either the classical (correlation of the wave functions in space and time) or the quantum senses (phase information). An example of collectivity is the hydrodynamic variables versus the micro-variables: the transition from kinetic theory to hydrodynamics is well known. The formal treatment refers to deriving the Naviers–Stoke equation from the BBKGY hierarchy. Here lies the relation of correlation and collectivity which is manifest in the micro-to-macro transition. Combined with the consideration of noise and decoherence above, we can see that the quantum/classical and the micro/macro transitions are interrelated issues.

3.2. Effective Theories: Renormalizability, Stochasticity, and Collectivity

The same factors arise in effective theories, which are theories valid at a lower energy or a larger scale, but constructed or derived from more fundamental theories for the more basic constituents. An example is the Fermi four-point interaction as a low-energy limit of the Weinberg–Salam electroweak interaction. Important issues are as follows: (1) Is the low-energy effective theory renormalizable, or effectively renormalizable— deeper understanding of effective theories has changed our view on renormalizability (see, e.g., ref. 70). (2) How do the effects of the high-energy sector or processes at a shorter length scale show up, if at all, at a larger scale in the low-energy observation range? (3) Usually the low-energy physics is described by a different set of variables from the high-energy physics—what we call the collective variables. How do we construct the collective state from the micro-physics? An even harder question: If we only know about the dynamics of the collective state, how much information about the deeper structure can we infer? (4) The interphase between high- and low-energy sectors can involve a crossover or a phase transition—what determines its character? In particular, fluctuations carry important information about the interphase and if it persists even in a miniscule amount, can provide valuable information about the short-scale behavior. In selected conditions such as in inflationary universe (like a 'zoom lense' [71]) or black holes (like a 'microscope' [72]) it offers hope to probe into sub-Planckian physics through structure formation [46] or Hawking radiation processes [73].

The above issues were phrased in a way which is particularly relevant to the search for a viable theory of quantum gravity from low-energy physics—by this we mean a quantum theory for the substructure of spacetime, not the quantization of general relativity. On the issue of stochasticity, Calzetta and Hu [67] studied an effective field theory and came up with a better understanding of the threshold behavior. We explored how the existence of a field with a heavy mass influences the low-energy dynamics of a quantum field with a light mass by expounding the stochastic characteristics of their interactions which take on the form of *fluctuations* in the number of (heavy field) particles created at the threshold, and *dissipation* in the dynamics of the light fields, arising from the backreaction of produced heavy particles. We claim that the stochastic nature of effective field theories is intrinsic, in that dissipation and fluctuations are present both above and below the threshold. Stochasticity builds up exponentially quickly as the heavy threshold is approached from below, becoming dominant once the threshold is crossed. But it also exists below the threshold and is in principle detectable, albeit strongly suppressed at low energies. The results derived here can be used to give a quantitative definition of the 'effectiveness' of a theory in terms of the relative weight of the deterministic versus the stochastic behavior at different energy scales.

In addition to stochasticity, one needs also to pay attention to two sets of issues: (a) How *collective variables* can be assigned for low-energy physics [74]. For gravity, if we assume that the metric or connection are the collective variables, how are they derivable from a deeper structure (e.g., strings) without our knowing the details of their interactions (e.g., string field theory)? (b) Viewing general relativity as the *hydrodynamic* *limit* of quantum gravity, examine the equivalent of the kinetic theory regime [66]. Work on the decoherent history of hydrodynamic variables [75] and on correlation history [21, 34] will be useful for pursuing these ideas. I will expand on the collectivity aspects in the next two sections before turning to the correlation aspect.[6]

4. GENERAL RELATIVITY AS GEOMETROHYDRODYNAMICS

In an essay written in 1996 [66] for the Second International Sakharov Conference, in the spirit of Sakharov's 'metric elasticity' idea [65], I presented the viewpoint that general relativity is hydrodynamics. It describes the collective state (call it 'spacetons'?) of a system of strongly interacting quantum objects (strings?) which span the spacetime substructure.

I examined the various conditions which underlie the transition from some candidate theory of quantum gravity to general relativity, specifically, the long-wavelength, low-energy (infrared) limits, the quantum-to-classical transition, the discrete-to-continuum limit, and the emergence of a macroscopic collective state from the microscopic consitituents and interactions of spacetime and fields. In the 'top-down' approach, I argued that nonequilibrium quantum field theory is needed to show how general relativity arises as various limits are taken in all candidate theories of quantum gravity, such as string theory, quantum geometry (via the Ashtekar spin connections or the Rovelli–Smolin loop representations), and simplicial quantum gravity. In the 'bottom-up' approach, which is the path have taken here, one starts with the semiclassical theory of gravity and examines how it is modified by graviton and quantum field excitations near and above the Planck scale. I mentioned three aspects based on recent findings of our work: (1) Emergence of stochastic behavior of spacetime and matter fields depicted by an Einstein–Langevin equation. The backreaction of quantum fields on the classical background spacetime manifests as a fluctuation-dissipation relation (discussed above). (2) Manifestation of stochastic behavior in effective theories below the threshold arising from excitations above. The implication for general relativity is that such Planckian effects, though exponentially suppressed, is in principle detectable at sub-Planckian energies [67]. (3) Decoherence of correlation histories and quantum-to-classical transition [21]. Following the observation of Gell-Mann and Hartle that the hydrodynamic variables which obey conservation laws are most readily decohered, one can [66], in the spirit of Wheeler [e.g.,

[6] The next two sections are excerpted from (the unpublished part of) an article [66]. Readers prone to be bored or annoyed by philosophical discourses should proceed to Section 6.

ref. 76] view the conserved Bianchi identity obeyed by the Einstein tensor as an indication that general relativity is a geometrohydrodynamic theory.

4.1. 'Top-Down': How to Reach the Correct Limits

The possible transitions we expect to find between quantum gravity and general relativity, i.e., quantum-to-classical transition, low-energy, long-wavelength (infrared) limits, discrete-to-continuum limit, extended structure to point structure, and micro/constituents versus macro/collective states, are manifest in varying degrees of transparency in three leading types of candidate theories of quantum gravity: the superstring theory [77] the loop representation of quantum geometry via spin connections [78], and simplicial quantum gravity [86]. In string theory, a spin-two particle is contained in the string excitations, and it is easy to see the limit taken from an extended structure to a point. The larger problem of how the target space (e.g., spacetime of 26 dimensions for bosonic string) can be deduced from, or at least treated on the same footing as, the world volume of fundamental branes, still remains elusive. The Bekenstein–Hawking expression for the black hole entropy [80] originally derived in semiclassical gravity is obtained as the tree-level result of many quantum theories of gravity [81].[7] But in the construction of a statistical mechanical entropy [83] from quantum field theory in curved spacetime, it is not so clear which of the many internal degrees of freedom of string excitations contribute to the leading quantum correction term. It is encouraging that recent advances in D-brane technology and duality relations have provided a statistical mechanical origin of black hole entropy from string theory, albeit so far only for near-extremal black holes [84]. This linkage with low-energy physics (semiclassical gravity results) will illuminate how the collective variables are chosen and the collective state formed. In the quantum relativity approach using Ashtekar's spin connection and the Rovelli–Smolin loop representation, the picture of a one-dimensional quantum weave behaving like a polymer is evoked [85]. When viewed at a larger scale the weaves appear to 'knit' a higher dimensional spacetime structure. This is an interesting picture, but how this collective process comes about—i.e., how the physical spacetime becomes a dynamically preferred entity and an infrared stable structure—remains to be explicated (cf. protein folding?). In simplicial quantum gravity [86], the classical limit might be obtained more easily in some versions (e.g., in the Ponsano–Regge $6j$ calculus [87], it is quite similar to the treatment of

[7] Jacobson [82] used the thermodynamic expression for black hole entropy to show how Einstein's equation can be derived as a thermodynamic equation of state. The underlying philosophy of this view is similar to ours.

ordinary spin systems via group-theoretic means, in place of the more involved considerations of environment-induced decoherence [11]), but essential properties like diffeomorphism invariance in the continuum limit are not guaranteed, such as in Regge calculus. The dynamical triangulation procedure (see refs. 88 for recent reviews) was believed to work nicely in these respects. But there are speculations that a first-order transition may arise which can destroy the long-wavelength niceties. How the general relativity limit comes about is not yet fully understood.

Many structural aspects of these theories in their asymptotic regimes (defined by the above-mentioned limits) near the Planck scale bear sufficient resemblance to the physics in the atomic and nuclear scales that I think it is useful to examine the underlying issues in the light of these better-understood and well-tested theories. These include on the one hand theories of 'fundamental' interactions and constituents, such as quantum electrodynamics (QED) and quantum chromodynamics (QCD)—add to them the well-developed yet untested theories of supersymmetry (SUSY) and grand unified theories (GUT)—which are indeed what piloted many of today's candidate theories of quantum gravity, and on the other hand theories about how these interactions and constituents manifest in a collective setting—theories traditionally discussed in condensed matter physics using methods of statistical mechanics and many-body theories. These two aspects are not disjoint, but are interlinked in any realistic description of nature [89, 74]. They should be addressed together in the search for a new theory describing matter and spacetime at a deeper level. The collective state description has not been emphasized as much as the fundamental interaction description. I call attention to its relevance because especially in this stage of development of candidate theories of quantum gravity, deducing their behavior and testing their consequences at low-energy constitute an important discriminant of their viability. Low-energy particle spectrum and black hole entropy are prime examples among the currently pursued topics.

Take, for example, the interesting observations related above, that four-dimensional spacetime is an apparent (as observed at low energy) rather than a 'real' (at Planck energy scale) entity—highlighted in Susskind's [90] world as hologram and 't Hooft's [91] view of the string-theoretic basis of black hole dynamics and thermodynamics. General relativity could be an emergent theory in some 'macroscopic,' averaged sense at the low-energy, long-wavelength limit. The fact that fundamental constituents manifest very different features at lower energies is not so surprising; they are encountered in almost all levels of structure—molecules from atoms, nuclei from quarks—referred to categorically as 'collective states.' How relevant and useful these variables or states are depends critically on the scale and nature of the physics one wants to probe. One cannot say that one is better than the other without stipulating the range of energy in question, the nature of the probe, and the precision of the measurement. Just as thermodynamic variables are powerful and economical in the description of long-wavelength processes, they are completely useless at molecular scales. Even in molecular kinetic theory, different variables (distribution and correlation functions) are needed for different ranges of interactions. In treating the relation of quantum gravity to general relativity it is useful to bear in mind these general features we learned from more familiar processes.

Even when one is given the correct theory of the constituents, it is not always an easy task to construct the appropriate collective variables for the description of the relevant physics at a stipulated scale. Not only are the derived structures different from their constituents, their effective interactions can also be of a different nature. There used to be a belief (myth) that once one has the fundamental theory, it is only a matter of details to work out an effective theory for its lower energy counterparts. Notice how nontrivial it is to deduce the nuclear force from quark–gluon interactions, despite our firm knowledge that QCD is the progenitor theory of nucleons and nuclear forces. Also, no one has been clever enough to have derived, say, elasticity from QED yet. Even if it is possible to introduce the approximations to derive it, we know it is plain foolish to carry out such a calculation, because at sufficiently low energy, one can comfortably use the stress and strain variables for the description of elasticity. (Little wonder quantum mechanics, let alone QED, is not a required course in mechanical engineering.)

4.2. 'Bottom-Up': Tell-Tale Signs from Low Energy

How the low-energy behavior of a theory is related to its high-energy behavior (issues of effective decoupling and renormalizability naturally would arise [92]), and whether one can decipher traces of its high-energy interactions or remnants of its high-energy components, have been the central task of physics since the discovery of atoms in the last century and subatomic particles in this century to today's probe on unified theories at ultrahigh energy. The symmetry of the particles and interactions existing at low energies are the only raw data we can rely on to construct (and appraise the degree of success of) a new unified theory. (Such is the central mission of, e.g., string phenomenology in reproducing the low-energy particle spectrum.) Some salient features of general relativity such as diffeomorphism invariance, Minkowski spacetime as a stable ground state, etc., are necessary conditions for any quantum theory of gravity to meet at the low-energy limit.[8] Approaching Planck energy from below, the beautifully simple yet deep theory of black hole thermodynamics [80] first discovered in semiclassical gravity is serving as a guide and providing a checkpoint for viable quantum gravity theories. Concerning the nature of the legacy (actually, the 'leftovers') from the physics at high energy, or special tell-tale signs at low energy, I would suggest paying careful attention to two features: topology and stochasticity. Topology refers to both nontrivial spactimes and field configurations, while stochasticity refers to the coarse-grained remnants of microphysics and fluctuation effects at the crossover. Here I will only focus on the latter feature, which is the central theme of stochastic gravity.

4.2.1. Fluctuations and Noise at the Threshold

An important feature of physics at the Planck scale depicted by semiclassical gravity is the backreaction of quantum effects of particles and fields, such as vacuum polarization and particle creation, on the classical gravitational spacetime. This is an essential step beyond classical relativity for the linkage with quantum gravity. For example, generalization to the $R + R^2$ theory of gravity is a necessary product from the renormalization considerations of quantum field theory in curved spacetimes. It should also be the low-energy form of string theory (plus dilaton and antisymmetric fields). Backreaction demands more, in that the quantum matter field is solved consistently with the classical gravitational field [2]. The consistency requirement in a backreaction calculation brings in two new aspects:

1. The classical gravitational field obeys a dynamics which contains a dissipation component arising from the backreaction of particle creation in the quantum field. The dissipation effect is in general nonlocal, as it is influenced by particle creation not only occurring at one moment, but also integrated over the entire history of this process [93, 3].

2. Creation of particles in the quantum matter field at the Planck energy (which is responsible for the dissipative dynamics of the gravitational field) can be depicted as a source which has both a deterministic and a stochastic component. The first part is the averaged energy density of created particles, which is known in previous treatments. The second part measures the differ-

[8] Note that if we view general relativity as a hydrodynamic theory in the same sense as the nuclear rotational and vibrational states in the collective or liquid drop model, we can see that as much as the symmetries of rotational and vibrational motion provide a useful description of the large-scale motion of a nucleus, they have no place in the fundamental symmetries of nucleons, much less their constituents, the quarks and gluons. In this sense one could also question the necessity and legitimacy of basic laws like Lorentz invariance and diffeomorphism invariance at a more fundamental level. It should not surprise us if they no longer hold for trans-Planckian physics.

ence of the amount of particles created in two neighboring histories and is depicted by a nonlocal kernel, the correlator of colored noise [18, 17]. The dissipation and noise kernels are related by a fluctuation-disspation relation. As described above, the backreaction equation is in the form of a Langevin equation, which we call the Einstein–Langevin equation [26, 25].

The Einstein–Langevin equation constitutes a new frontier for us to explore possible phase transition and vacuum instability issues, which I believe many of the 'top-down' approaches would also encounter in this crossover regime.

4.2.2. Stochastic Behavior Below the Threshold

What are the tell-tale signs for a low-energy observer of the existence of a high-energy sector in the context of an effective field theory? It helps to adopt an open system viewpoint to consider effective theories and explore their statistical mechanical properties. The question is to compare the difference between a theory operative (i.e., giving an adequate description) at low energies (as an open system, with the high-energy sector acting as the environment) to an exact low-energy theory taken as a closed system. We know that there are subtle differences between the two, arising from the backreaction of the heavy on the light sector. Though not obvious, the stochastic behavior associated with particle creation above the threshold (which for gravitational processes is the Planck energy) is related to the dissipative behavior of the background spacetime dynamics. (This has been known for some time; see, e.g., ref. 5.) Schwinger's result [94] for pair production in a strong electromagnetic field is a well-known example. This effect at very low energy has been ignored, however, as it is usually regarded as background noise covered by very soft photons. That such a noise carries information about the field at high energy was only pointed out recently [67]. Using a simple interacting field model, Calzetta and I found that even at energy way below the threshold, stochastic effects, albeit at extremely small amplitudes, can reveal some general (certainly not the specific) properties of the high-energy sector. Finally one can also show from the decoherence aspects of quantum theories in reaching their classical limits [66] why general relativity can be viewed as the hydrodynamic limit of quantum gravity.

5. LOW-ENERGY COLLECTIVE-STATE PHYSICS AND BEYOND

Suppose one takes this viewpoint seriously, what are the possible implications? One can make a few general observations here.

5.1. Quantizing Metric May Yield Only Phonon Physics

First, the laws governing these collective variables are classical, macroscopic laws. It may not make full sense to assume that by quantizing these variables directly one would get the micro-quantum basis of the macro-classical theory, as has been the dominant view in quantum gravity. Just as the energy density ρ and momentum densities p in the Einstein equation are the hydrodynamic variables of a matter field, quantization should only be performed on the microscopic fields $\Phi(x)$ from which they are constructed. If one did so for the metric or the connection variables, one would get the quantum excitations of geometry in the nature of phonons in relation to atoms (or other quantum collective excitations in condensed matter physics). That may be the next order of probe for us, and may reveal some interesting phenomena, but it is still very remote from seeing the nucleon structure in the solid lattice or the attributes of quantum electrodynamics. In the analogy mentioned above, we do not expect quantum elasticity to tell us much about QED.

Second—and this is perhaps the more interesting aspect—assuming that the metric and connections are the collective variables, from the way they are constructed, what can one say about their microscopic, quantum basis? Historically this question was asked repeatedly when one probed from low- to high-energy scales, trying to decipher the microscopic constituents and laws of interactions from macroscopic phenomena. This is like going from phonons to the structure of atoms, from nuclear rotational spectrum to nucleon strucuture—not an easy question to answer. But there are nevertheless ways to guide us, e.g., in terms of some tell-tale signs. In the above analogies, recall that atomic spectroscopy reveals many properties about the electron–electron and electron–nucleus interactions, low-temperature anomalous behavior of specific heat reveals the quantum properties of electrons, and the intermediate boson model bridges the symmetry of the collective modes with that of the independent nucleons. To address questions like these, one needs to proceed from both ends: One needs to postulate a theory of the microscopic structure, and work out its collective states at large scale and low energies. One also needs to consider the consequences of the known low-energy theory, paying attention to subtle inconsistencies or mistakenly ignored trace effects from higher energy processes. Indeed, this is what is going on today, with string theory as the micro theory, and semiclassical gravity and particle phenomenology as its low-energy limit. The viewpoint I am proposing would suggest focusing on collective states (solitons?) of excitations of the fundamental string on the one hand and a detailed study of the possible new phenomena in quantum field theory in curved spacetime on the other, such as fluctuations and phase transitions around the Planck energy, quantum corrections to the black hole entropy, resonance states, and spacetime foams.

5.2. Common Features of Collective States Built from Different Constituents

As mentioned above, there are two almost orthorgonal perspectives in depicting the structure and properties of matter. One is by way of its constituents and interactions, the other according to its collective behavior. The former is the well-known and well-trodden path of discovery of QED, QCD, etc. If we regard this chain of QED-QCD-GUT-QG as a vertical progression depicting the hierarchy of basic constituents, there is also a horizontal progression in terms of the stochastic-statistical-kinetic-thermodynamic/hydrodynamic depiction of the collective states. It should not surprise us that there exist similarities between matter in the same collective state (e.g., hydrodynamics) but made from different constituents. Macroscopic behavior of electron plasmas is similar in many respects to the quark–gluon plasma. Indeed, one talks about magnetohydrodynamics from Maxwell's theory as well as magnetochromohydrodynamics from QCD. In this long-wavelength, collision-dominated regime, they can both be depicted by the hydrodynamics of fluid elements, which is governed simply by Newtonian mechanics. The underlying micro-theories are different, but the hydrodynamic states of these constituents are similar. Here I am proposing that general relativity being a hydrodynamic limit (of some candidate theory of quantum gravity) is an effective theory in the way that nuclear physics is with regard to QCD, and atomic physics is with regard to QED. They are all low-energy collective states of a more fundamental set of laws and can share some similarities. The macroscopic, hydrodynamic equations and their conservation laws like the Navier–Stokes and the continuity equations are all based on dynamical and conservation laws of microphysics (e.g., Newtonian mechanics), but when expressed in terms of the appropriate collective variables, they can take on particularly simple and telling forms. Thermodynamic variables like temperature, entropy, etc. (think black hole analogy—mass, surface area) are derived quantities with their specific laws (three laws) traceable via the rules of statistical mechanics (of Gibbs and Boltzmann) to the laws of quantum mechanics. Rules of statistical mechancis are important when we probe a deeper layer of structure from known low-energy theories such as semiclassical gravity: we need to know how to disentangle the collective states in order to see how the microphysics works.[9] It is hard to imagine how a complete theory of microphysics can be attained without going through this step.

[9] Savor the importance of, say, coming up with a statistical mechanical definition of temperature in a canonical ensemble as the rate of change of the accessible states of a system in contact with a heat reservoir with respect to changes in energy, and we can appreciate the importance of Gibbs' work in relation to quantum physics.

5.3. Hydrodynamic Fluctuations Applied to Black Holes and Cosmology

A problem where this analogy with collective models may prove useful is that of black hole entropy. If we view the classical expression for black hole entropy to be a hydrodynamic limit and the corrections to it as arising from hydrodynamic fluctuations, we could use linear response theory to approach conditions near thermodynamic equilibrium and construct a non-equilibrium theory of black hole thermodynamics.[10] It also seems to me that many current attempts to deduce the quantum corrections of black hole entropy from the micro-quantum theory of strings could be missing one step. This is like the correpondance between results predicted from the independent particle (nucleon) model (where one can construct the shell structures) and that from the liquid drop model (where one can construct the collective motions)—a gap exists which cannot easily be filled by simple extensions of either model operative in their respective domains of validity. This involves going from the individual nucleon wavefunctions to the collective states of a nucleus. It is likely that only specific appropriate combinations of fundamental string excitation modes which survive in the long-wavelength limit can contribute to the excitations of the collective variables (area and surface gravity of black hole) which enter in the (semiclassical gravity) black hole entropy.[11]

Viewing classical GR as hydrodynamics implies that stochastic gravity and Einstein–Langevin equation would depict the hydrodynamic fluctuations of spacetime dynamics as induced by quantum field processes at the Planck scale. One could study the behavior of metric and field fluctuations with this Langevin equation in a way similar to that of critical dynamics for fluids and condensed matter.

In summary, note that the progress of physics—the probing of the structure and dynamics of matter and spacetime—has always moved in the direction from low to high energies. One needs to pay attention to the seemingly obvious facts at low energies and probe any discrepancy or subtleties not usually observed to find hints to the deeper structures. By examining how certain common characteristics of all successful low-energy theories (here, I only discuss the hydrodynamic and thermodynamic aspects) may

[10] The black hole backreaction problem has been studied by many authors, notably York [55] and Anderson *et al.* [48]. We are taking a nonequilibrium statistical field theory approach. We aim to get the fluctuations of the energy-momentum tensor of a quantum field in a perturbed Schwarzschild spacetime [38], examine how they might induce dissipations of the event horizon, and deduce a susceptibility function of the black hole. This would realize the proposal of Sciama that a black hole in equilibrium with its Hawking radiation can be depicted as a quantum dissipative system [58] (see also ref. 95).

[11] This statement made in July 1996 should be viewed in the context of new developments since then in the statistical mechanical origin of black hole entropy via D-branes [84].

recur in a new theory at a higher energy, and how they differ, we can perhaps learn to ask the right questions and focus on some hitherto neglected aspects.

6. QUANTUM MICRODYNAMICS VIA CORRELATION HIERARCHY

In the last two sections we touched on two issues deemed important in the transition between quantum and classical gravity, i.e., stochasticity (or fluctuations) at the intermediate regime (stochastic semiclassical gravity) and collectivity at the low-energy (general relativity) regime. We now focus on the correlation aspect, as a way for probing the full quantum regime. Along the way we will mention a few problems which may shed light on the passage from stochastic to quantum gravity.

6.1. Correlation and Coherence

If we look back at the equations in Section 1.4 and compare the semiclassical (sC), stochastic (St), and quantum (Q) regimes, we see first that in the sC case, the classical metric correlations are given by the product of the vacuum expectation value of the energy-momentum tensor, whereas in the quantum case the quantum average of the correlation of metric (operators) is given by the quantum average with respect to the fluctuations in both the matter and the gravitational fields. In the stochastic case the form is closer to the quantum case except that now the quantum average is replaced by the noise average, and the average of the energy-momentum tensor is taken with respect only to the matter field. The important improvement over the semiclassical case is that it now carries information on the correlation of the energy-momentum tensor of the fields and its induced metric fluctuations. This is another way to see why the stochastic description is closer to the quantum truth. More intuitively, the difference between quantum and semiclassical conditions is that the latter loses all the coherence in the quantum gravity sector. The stochastic improves on the semiclassical condition in that partial information related to the coherence in the gravity sector is preserved as reflected in the backreaction from the quantum fields and manifests as induced metric fluctuations. That is why we need to treat the noise terms with maximal respect. They contain quantum information absent in the classical condition. The coherence in the geometry is related to the coherence in the matter field, as the complete quantum description should be given by a coherent wave function of the combined matter and gravity sectors. Since the degree of coherence can be measured in terms of correlations, our strategy is to examine the higher correlations of the matter field, starting with the variance of the energy-momentum tensor in order to probe into or retrieve whatever partial coherence remains in the quantum gravity sector. The noise we worked out in the Einstein–Langevin equation above contains the fourth-order correlation of the quantum field (or gravitons when considered as matter source) and manifests as induced metric fluctuations. Let us see what can be done to get closer to the quantum picture.

If we view classical gravity as an effective theory, i.e., the metric or connection functions as collective variables of some fundamental particles which make up spacetime in the large and general relativity as the hydrodynamic limit, we can also ask if there is a midway station like kinetic theory from molecular dynamics, from quantum microdynamics to classical hydrodynamics. This transition involves both the micro-to-macro transition and the quantum-to-classical transition, which is what constitutes the mesoscopic regime for us.

For our present purpose we can represent quantum gravity as an interacting quantum field (of fermions?) and we shall traverse this passage using the correlation dynamics from the (nPI) master effective action. There are two aspects in this problem: coherence of a field as measured by its correlation (for quantum as well as classical), and quantum-to-classical transition. We wish to treat both aspects with a quantum version of the correlation (BBGKY) hierarchy, the Schwinger–Dyson equations. There are three steps involved: First, show how to derive the kinetic equations from quantum field theory, or to go from Dyson to Boltzmann [96]. Second, show how to introduce the open system concept to the hierarchy. For this we need to introduce the notion of 'slaving' in the hierarchy, which renders a subset made up of a definite number of lower order correlation functions as an effectively open system, where it interacts with the environment made up of the higher correlation functions. Third, show why there should be a stochastic term in the Boltzmann equation when contributions from the higher correlation functions are included.

6.2. Kinetic Field Theory via Master Effective Action

The first step was taken in the 1980's, when Calzetta and Hu [96], among others [97] (see ref. 98 for earlier work and ref. 99 for recent developments) showed how the quantum Boltzmann equation arises as a description of the dynamics of quasiparticles in the kinetic limit of quantum field theory. The main element in the description of a nonequilibrium quantum field is its Green functions, whose dynamics is given by the Dyson equations. For the second step, we showed in 1995 [34] how the Schwinger–Dyson equations can be obtained from an ∞PI master effective action and how the coarse-grained (truncation with slaving) n-point correlation functions behave like an effectively open system. Recently [35] we have taken the third step in identifying such a noise term in the Boltzmann equation (its classical limit reproduces the result of Kac and Logan [100]) and proving a fluctuation-dissipation relation for these correlation noises. The resultant stochastic Boltzmann equation has features of both the Langevin and Boltzmann equations. With this one can then begin to investigate the possibility of using the correlation hierarchy to infer the quantum microdynamics. For illustration, I will just show the lowest order in the correlation hierarchy by way of the master effective action.

The mean-field and the two-point function which one uses to deduce kinetic theory or critical dynamics results are just the lowest two elements in the full Schwinger–Dyson (SD) hierarchy of correlation functions. In general the complete set is required to recover full (including phase) information in a quantum field. If we now view the problem in this light, we can see how dissipation and fluctuations arise when the hierarchy is truncated and the higher correlations are slaved (I refer to these two procedures as coarse-graining), in the same way as the Boltzmann equation is derived from the BBGKY hierarchy. What is new in our current understanding is that there should also be a noise term in addition to the collision term in the Boltzmann equation.

In ref. 34 we showed how this hierarchy of SD equations can be derived from the master (∞PI) effective action, so here I will just show the form of the 2PI. For a scalar field $\Phi(x)$ with classical action $S[\Phi]$ under an external source $J(x)$ the generating functional $W[J]$ is given by [101]

$$\exp\{iW[J]\} = \int D\Phi \exp\{iS[\Phi] + i\int d^4x\, J(x)\Phi(x)\} \tag{41}$$

from which one can obtain the expectation value or mean field

$$\phi(x) = \frac{\delta W}{\delta J}\bigg|_{J=0} \tag{42}$$

The effective action is the Legendre transform of W,

$$\Gamma[\phi] = W[J] - \int d^4x\, J(x)\phi(x) \tag{43}$$

from which we obtain the equation of motion

$$\frac{\delta\Gamma}{\delta\phi} = 0 \tag{44}$$

In a causal theory, we must adopt Schwinger's CTP formalism. The point x may therefore lie on either branch of the closed time path

(a, b = $\pm$), or equivalently we may have two background fields $\phi^a(x) = \phi(x^a)$. The classical action is defined as

$$S[\phi^a] = S[\Phi^1] - S[\Phi^2]^* \tag{45}$$

which automatically accounts for all sign reversals. We also have two sources

$$\int d^4x\, J_a(x)\Phi^a(x) = \int d^4x\, [J^1(x)\Phi^1(x) - J^2(x)\Phi^2(x)]$$

and obtain two equations of motion

$$\frac{\delta\Gamma}{\delta\phi^a} = 0 \tag{46}$$

These equations always admit a solution where $\phi^1 = \phi^2 = \phi$ is the physical mean field. After this identification, they become a real and causal equation of motion for ϕ.

The functional methods we have used so far to derive the dynamics of the mean field may be adapted to investigate more general operators. In order to find the equations of motion for two-point functions, for example, we add a nonlocal source $K_{ab}(x, x')$ [102, 96]

$$\exp\{iW[J_a, K_{ab}]\} = \int D\Phi^a \exp i\left\{S[\Phi^a] + \int d^4x\, J_a\Phi^a + \frac{1}{2}\int d^4x\, d^4x'\, K_{ab}\Phi^a\Phi^b\right\} \tag{47}$$

It follows that

$$\frac{\delta W}{\delta K_{ab}(x, x')} = \frac{1}{2}\,[\phi^a(x)\phi^b(x') + G^{ab}(x, x')]$$

Therefore the Legendre transform, the so-called 2PI effective action,

$$\Gamma[\phi^a, G^{ab}] = W[J_a, K_{ab}] - \int d^4x\, J_a\phi^a - \frac{1}{2}\int d^4x\, d^4x'\, K_{ab}[\phi^a\phi^b + G^{ab}] \tag{48}$$

generates the equations of motion

$$\frac{\delta\Gamma}{\delta\phi^a} = -J_a - K_{ab}\phi^b; \qquad \frac{\delta\Gamma}{\delta G^{ab}} = -\frac{1}{2}K_{ab} \tag{49}$$

This is an ($n = 2$) example of the nPI effective action. When $n \to \infty$, this is known as the master effective action (MEA). The master effective action is a functional of the whole string of Green functions of a field theory whose variation generates the Schwinger–Dyson hierarchy. In ref. 34 we (1) gave a formal construction of the master effective action, (2) showed how truncation in *n*PI is related to loop expansion, and (3) showed how 'slaving' leads to dissipation.

6.3. Theoretical Considerations: *n*PI, 1/*N*, and Loop Expansions

In the above we defined the master effective action and showed its relation to the Schwinger–Dyson hierarchy. From this one can establish a kinetic theory of nonlinear quantum fields, derive the kinetic equations [96], and a correlation noise arising from the slaving of the higher correlation functions. The stochastic Boltzmann equation [35] contains features which would enable us to make connection with the stochastic equation in semiclassical gravity. This comes about from the following consideration: The Boltzmann equation describes the evolution of a one-particle distribution function driven by a two-particle collision integral, and the stochastic Boltzmann equation incorporates the contribution of a higher order correlation function. The Langevin equation was derived in the framework of an open system, the noise arising from coarse-graining the environment. Truncation and slaving as carried out in the hierarchy yield an effectively open system and the master effective action leads to the stochastic Boltzmann equation similar to the Langevin equation in an open system. (From here we can see at work the two major paradigms in nonequilibrium statistical mechanics: the Boltzmann–BBGKY and the Langevin–Fokker–Planck descriptions.) The corresponding situation for interacting quantum fields can be applied to quantum gravity—assuming that it can be represented by some interacting quantum field—and illuminate how one should proceed from the standpoint of stochastic gravity. We can get a handle on the correlation of the underlying field by examining the hierarchy of equations, of which the Einstein–Langevin equation describes only the lowest order correlations: the relation of the mean field to the two-point function, and the two-point function to the four-point function (variance in the energy-momentum tensor). One can in principle move higher in this hierarchy to decipher the higher correlation contributions. Notice that we have only dealt with the correlation aspect, the quantum-to-classical aspect remains. This can be treated by the decoherence of correlation histories discussed earlier [21].

While we are discussing formal matters, I should mention that it is worthwhile to also include the large-*N* expansion for comparison. There exists a relation between correlation order and the loop order [34]. One can also relate it to the order in the large-*N* expansion. It has been shown that the leading-order $1/N$ expansion for an N-component quantum field yields the equivalent of semiclassical gravity [103]. The leading-order $1/N$ approximation yields mean-field dynamics of the Vlasov type [104], which shows Landau damping which is intrinsically different from the Boltzmann dissipation. In contrast, the equation obtained from the nPI (with slaving) contains dissipation and fluctuations manifestly. It is apparent that the next to leading order incorporates interactions corresponding to coherent scattering of particles. It would be of interest to think about the relation between semiclassical and quantum conditions in the light of the higher $1/N$ expansions, which is quite different from the scenario associated with the correlation hierarchy.

6.4. Physical Considerations: Strongly Correlated Systems

At this point it is perhaps useful to bring back the opening theme of our discussion, i.e., semiclassical gravity as mesoscopic physics, and examine similar concerns.

To practitioners in condensed matter and atomic/optical physics, mesoscopia refers to rather specific problems where, for example, the sample size is comparable to the probing scale (nanometers), or the interaction time is comparable to the time of measurement (femtoseconds), or the electron wavefunction correlated over the sample alters its transport properties, or the fluctuation pattern is reproducible and sample-specific. Take quantum transport. Traditional transport theory applied to macroscopic structures is based on kinetic theory, while that for mesoscopic structures is usually based on near-equilibrium or linear response approximations (e.g., Landauer–Bütiker formula). New nanodevice operations involve nonlinear, fast-response, and far-from-equilibrium processes which are sensitive to the phases of the electronic wavefunction over the sample size. These necessitate a new microscopic theory of quantum transport. One serious approach is using the Keldysh method in conjunction with Wigner functions (e.g., ref. 105). It is closely related to the closed-time-path formalism we developed for nonequilibrium quantum fields aimed at similar problems in the early universe and black holes [96].

Now focusing on the issue of correlations and quantum coherence while using the analogy with mesosystems, we see that what appears on the right-hand side of the Einstein–Langevin equation—the stress-energy two-point function—is analogous to conductance, which is given by the current–current two-point function. What this means is that we are really calculating the transport function of (the matter particles as depicted by) the quantum fields. Following Einstein's keen observation that spacetime dynamics is determined by (while it also dictates) the matter (energy density), we expect that the transport function represented by the current correlation in the matter (fluctuations of the energy density) would also have a geometric counterpart and equal significance at a slightly higher energy scale. The hydrodynamic analogy given earlier also makes sense here: Conductivity, viscosity, and other transport functions are hydrodynamic quantities. For many practical purposes we do not need to know the details of the fundamental constituents or their interactions to establish an adequate depiction of the low- or medium-energy physics, but can model them with semiphenomenological concepts (like mean free path and collisional cross sections). In the mesoscopic domain the simplest kinetic model of transport using these concepts is no longer accurate. One needs to work with system–environment models and keep the phase information of the collective electron wavefunctions. When the interaction among the constituents gets stronger, effects associated with the higher correlation functions of the system begin to show up. Studies in strongly correlated systems are revealing in these regards [68, 105]. For example, fluctuations in the conductance—from the four point function of the current—carry important information such as the sample-specific signature and universality. Although we are not quite in a position, technically speaking, to calculate the energy-momentum four-point function, thinking about the problem in this way may open up many interesting conceptual possibilities, e.g., what does universal conductance fluctuations mean for spacetime and its underlying constituents? In the same vein, I think studies of nonperturbative solutions of gravitational wave scattering [106] will also reveal interesting information about the underlying structure of spacetime (beyond the hydrodynamic realm). Thus, viewed in the light of mesoscopic physics, with stochastic gravity we are really begining to probe into the higher correlations of quantum matter and with them the associated excitations of the collective modes in geometrohydrodynamics.[12]

7. TOWARD QUANTUM GRAVITY

I now integrate what I discussed in the above and enumerate possible activities at the Planck scale, related to the three aspects of fluctuations, correlation, and collectivity.

[12] Of course, walking along this pathway, it will still take a while before one sees the microscopic quantum picture—the constituents of spacetime, like electrons in quantum transport. One may indeed never see it, because one needs to seek a set of variables for the basic constituents different from those for the collecive modes. But as far as what low-energy observers can decipher, these collective phenomena are all that one can observe and the hydrodynamic quantities such as the transport functions and their derived constructs actually offer a better set of variables for their description since the equations and the physics are simpler.

7.1. Quantum Tunneling, Particle Creation, and Phase Transition at the Planck Scale

The Langevin equation description of semiclassical gravity opens up a new horizon at the juncture of general relativity and quantum gravity theories in that it enables one to examine the properties of fluctuations in the quantum matter fields and their effect on the stability of the classical spacetime structure.

At the Planck scale when quantum effects of gravity become significant, physical laws as well as the structure of spacetime and matter may undergo fundamental changes in form and content. Many such changes could be the outcome of phase transitions. The study of Planck-scale phase transitions is thus of fundamental theoretical value. Near the Planck time when the gravitational field is strong and when spacetime geometry changes drastically, vacuum particle production is abundant, and any phase transition would likely be accompanied by particle production. In treating Planck-scale phase transitions, not only is the effective potential ill defined, because the background field changes in time, but the background field splitting often assumed in the derivation of the effective Lagrangian would become ineffective (because the background field can change as much as, and as fast as, the fluctuation fields). In such cases (or in cases where global properties of spacetime like boundary or topology are involved), one would need to use nonperturbative methods such as instanton solutions (in Euclidean formalism). However, to incorporate statistical processes one needs a real-time description. It is difficult to join these two worlds, but if we can (say, using a new route via Langevin or Fokker–Planck or master equations), we will be able to deal with a wider range of issues. Phase transition in the form of spinodal decomposition as applied to defect formation is currently under investigation [107]. Here I would like to comment on the process of nucleation via quantum tunneling in relation to stochastic gravity.

7.1.1. Tunneling and Particle Creation as Vacuum Decay

My view is that both tunneling and particle creation are manifestations of vacuum instability, but with different setup of boundary conditions for these two processes. (The tunneling probability and the probability for finding a pair of particles created are both given by the imaginary part of the effective action.) While in a tunneling problem the system transits from one definite (metastable) state to another, in particle production (from dynamic spacetimes) it is a continuous change from an initial vacuum to a final vacuum, with inequivalent Fock spaces at all intermediate states. One can formulate this problem first in the setting of quantum mechanical potential scattering using Bogolubov transformations, and then in the effective action formalism via vacuum persistence amplitudes. The advantage of this unifying view is that many aspects of tunneling can be addressed by established methods of treating particle creation.

7.1.2. *Tunneling with Particle Creation: Dynamics and Dissipation*

If particle production occuring during tunneling is not strong enough to disrupt the tunneling process, one can treat this as a test-field problem. Rubakov [108] first attempted this problem with a nonunitary Bogolubov transformation (I do not find this method so agreeable; see also criticism by Vachaspati and Vilenkin [109]). I prefer to use a real-time approach and treat particle creation in the fluctuation fields as parametric amplification by the background field (as one encounters in the postinflation reheating problem [110]). If the particle creation is so strong as to alter the tunneling process, one needs to take the backreaction into consideration and solve the 'dynamics' of tunneling and particle creation self-consistently. Since particle creation can be viewed as a dissipative process, this becomes a problem of tunneling with dissipation [111]. One can apply stochastic field theory for its treatment where dissipation and noise are manifest. Insofar as particle creation is a form of amplified quantum noise, the interesting processes of stochastic resonance and noise-induced transitions could also shed light on this issue.

7.1.3. *Tunneling and Decoherence in Quantum Cosmology*

Vilenkin has proposed a tunneling boundary condition in quantum cosmology in the so-called 'birth of the universe' scenario [112]. What is the effect of particle creation on the tunneling wavefunction? Would dissipation terminate the tunneling process and give a 'still birth' of the universe? Does it make sense to talk about matter 'before' (Euclidean time!) the universe? One can investigate this issue in the context of minisuperspace quantum cosmology by studying the effect of dynamics on quantum fluctuations (of matter fields and spacetimes) during tunneling. A related problem is decoherence and tunneling: Could vacuum fluctuations induce a quantum-to-classical transition in the tunneling wave function of the universe, giving rise to a semiclassical regime with desirable attributes which could generate our own universe, or will dissipation alter the picture irrevocably? One can incorporate results on dissipative tunneling into earlier studies of decoherence with backreaction in quantum cosmology (e.g., Paz and Sinha [6]). In adopting the influence functional scheme, one would be working with the density matrix of the universe, and the propagators of the reduced density matrix would be replacing the \$ matrix of Hawking and Page [113, 114] (similar in-out and in-in boundary condition difference would matter). This would also offer a new angle toward the issues of unitarity and information loss in quantum gravity.

7.1.4. Tunneling with Topology Change

Just as particle creation occurs when vacuum fluctuations of a quantum field get strong, pair creation of black holes may become important when metric fluctuations are large. One expects topology change in the spacetime to occur at the Planck energy via tunneling. This is also part of the activity in a spacetime foam which has been studied by Hawking and his associates for a long time (see references in Section 7.5). In approaching these problems usually one defines the end states in terms of Lorentzian geometry and describes the tunneling process by the Euclidean instanton method. Finding the joining solution between two end states is not simple, though, as it is not so well defined.

Similar to particle creation, one may expect to cast black hole pair creation as a dissipative process. If so, one would also need to work in real-time dynamics. The backreaction of these pair creation processes is expected to be strong at the Planck energy. So the same set of issues will arise as before. For example, how would pair production of particles and black holes associated with topology change alter the tunneling rate and the topology change itself? Our current understanding has not reached this level of sophistication, but these are important issues to think about.

7.2. Nucleation of Black Holes from Curved Spacetime and Growth of Fluctuations and Forms

From earlier discussions we see that vacuum instability and phase transition may play an important role in revealing the structure of spacetime at the Planck scale. Ideally we wish to first formulate a quantum field-theoretic description of nucleation problem for first-order phase transitions in general, and then examine specific and related problems in gravity such as nucleation of black holes from hot flat space [61] or black hole pair creation in a de Sitter universe [125]. The first problem was studied by a number of authors in the 1980s [61, 115, 116] using Euclidean instanton methods to calculate the probability of nucleation. If one could cast this problem in the form of a Langevin or Fokker–Planck equation, one could reexamine this process as a dynamical critical phenomenon in real time. Similarly we wish to carry out a first-principles quantum field-theoretic description of spinodal decomposition for the second-order phase transitions. We have just started this latter project with application to defect formation in the early universe [107].

Advances in far-from-equilibrium sciences in the last decade show that correlations and noise in nonlinear systems are responsible for a great variety of structures and forms [117, 118]. Planck-scale fluctuations can be the germinating source for large-scale structures in the universe. Noise-induced phase transition is an important class of problems originally studied by Kramer for chemical kinetics. It is now applied with techniques from stochastic gravity by Calzetta and Verdaguer [40] to the early universe. They found the probability of a universe making such a phase transition to be very close to that of quantum tunneling studied earlier by Vilenkin [112] in the so-called 'birth of the universe from nothing' scenario. I only wish to add one observation. The proximity of these two results appears to me not a plain accident. Let us ponder the relation between noise-induced transition versus quantum tunneling. While the former usually refers to thermal noise (at finite temperature) in an environment, the latter refers to quantum noise (vacuum fluctuations). Even though one does not stipulate an environment for quantum tunneling, quantum fluctuations are ubiquitous and free (not quite: they are attached as the coarse-grained leftovers from activities in the high-energy sector, as reflected in some generalized uncertainty principle). So the real difference is between thermal and vacuum fluctuation-induced effects. The relation between these fluctuations in terms of their effect on decoherence has been studied [119, 120] in the context of finding an uncertainty relation at finite temperature. Looking at the problem in another way, in terms of the correlation hierarchy, quantum mechanical description invokes only the lowest order correlations. At higher energy or with finer resolutions, higher order correlations will partake more in the tunneling or transition process. One can use the Schwinger–Dyson hierarchy and correlation noise to put quantum and thermal fluctuations on the same footing. Neither of them needs an environment nor a temperature stipulation. If we can relate these two classes of processes (quantum and statistical mechanical) we may find a way to deal with particle creation and tunneling together—quantum or noise-induced—in a unified real-time formalism.

7.3. Wave Propagation in Random Geometry and Simplicial Gravity

In a recent paper Hu and Shiokawa [121] studed some novel effects associated with electromagnetic wave propagation in a Robertson–Walker universe and the Schwarzschild spacetime with a small amount of metric stochasticity. By showing the formal equivalence of the wave equations in curved spacetimes with (flat space) wave propagation in a material medium and identifying the dependence of the refractive index on the metric components, one can introduce metric fluctuations as a stochastic component in the permittivity function and borrow the insights from known results of wave propagation in random media. We find that localization of electromagnetic waves occurs in a Robertson–Walker universe with time-independent metric stochasticity, while time-dependent metric stochasticity induces exponential instability in the particle production rate. For the Schwarzschild metric, time-independent randomness can decrease the total luminosity of Hawking radiation due to multiple scattering of waves outside the black hole and gives rise to event horizon fluctuations and thus fluctuations in the Hawking temperature.

In that work the source of metric stochasticity is represented by a stochastic component in the permitivity function. It is desirable to give a microscopic derivation of metric stochasticity. Stochastic components in the metric can be induced by primordial gravitational waves, topological defects on the sub-Planckian scale, or intrisic metric fluctuations of background spacetimes at the Planck scale. We should be able to calculate these components with the help of stochastic gravity. Their detection and analysis can provide valuable information about the state of the early universe and black holes. After this one can probe wave propagation in random geometry itself [88] via random potentials. Eventually one should connect this to simplicial gravity [86]. In addition to seeking the continuum limit from discrete geometries, it is of interest to examine if a possible disorder–order transition can arise from stochastic spacetimes, and whether one could use this to divide the effective (low-energy, ordered, or smoothed-out phase of) spacetime into universality classes.

7.4. Planck-Scale Resonance States

Following the progression from hydrodynamics to kinetic theory and quantum microdynamics, one may ask if there could exist quasistable structures at energy scales slightly higher than (or observation scales finer than) the semiclassical scale. Assuming that string theory is the next-level micro-theory, do there exist quasistable structures between that and general relativity? This is like the existence of resonance states (as quasistable particles) beyond the stable compounds of quarks (baryons) or quark–antiquarks (mesons). Viewed in the conceptual framework of kinetic theory, there could exist such states if the interparticle reaction times (collision and exchange) and their characteristic dynamics (diffusion and dissipation) become commensurate at some energy scale. (Turbulence in the nonlinear regime could show up in these intermediate state.) In the framework of decoherent history discussed above, it could also provide metastable quasiclassical structures. It would be interesting to find out if such structures can in principle exist around the Planck scale. This question is stimulated by the hydrodynamic viewpoint, but the resolution would probably have to come from a combination of efforts from both the top-down and the bottom-up approaches. Deductions from high-energy string theories would also benefit from knowing what different collective states are likely to exist in the low-energy physics of general relativity and semiclassical gravity.

7.5. Spacetime Foams

The beautiful and alluring ideas of Wheeler [42] on metric fluctuations and spacetime foams have only seen intermittent meaningful developements in the last 35 years since it was conceived, foremost by Hawking and his associates. Hawking's work on quantum gravitational bubbles [122], wormholes and baby universes [123], virtual black holes [124], and black hole pair creation [125] provided a solid base for such inquires. At the Planck scale geometric and topological fluctuations of spacetime are expected to be important. At a scale close to but larger than the Planck scale, stochastic gravity can provide a good physical depiction. The extensively developed tools and concepts there can help one treat the coarse-grained state of these 'building blocks' of spacetime foams and come up with quantitative descriptions and predictions for low-energy phenomenology. Metric fluctuations induced by quantum matter fields (including gravitons) in the backreaction problems we have studied so far offer perhaps the simplest and the most ubiquitous type of ingredient in the spacetime foam. We know them quantitatively by the noise or the correlation functions (see examples given at the beginning for weakly inhomogeneous cosmological spacetimes and far-field thermal black hole background). The use of open system concepts enables one to view them as thermal baths [126] in the most naive approximation such as in the Fokker–Planck limit (Markovian behavior at high-temperature Ohmic bath in the case of bilinear coupling between the system and bath), but one lesson we learned from stochastic gravity is that these 'noises' are by no means trivial, as they contain precious information about the substructures and their constitution at a higher energy level. It would be interesting to examine the low-energy remnants of the other types of spacetime foams mentioned above. If we view them as an environment interacting with the classical geometry (which actually is the mean value taken with respect to all possible stochastic source distributions) and study their behavior with the right model in nonequilibrium statistical mechanics, we can get a rich physical picture with quantitative information (dissipation, diffusion, correlation, decoherence). For example, virtual black holes can, according to a recent suggestion [126], be representated at low energy by effective bilocal couplings. Hawking *et al.* reasoned that spacetime is made up of three kinds of basic building blocks of topological class, $S^2 \times S^2$, K^3, and CP^2, and gravitational bubbles are believed to be their quantum fluctuations. It is not easy to deal with these topological fluctuations, but in an effective description the vertex for the bubble scattering can be viewed as arising from the exchange of a very large number of gravitons. From this one can construct an open system model for multigraviton exchange and come up with a stochastic gravity version of this type of spacetime foam contribution. Wormholes are more complicated, as they are multiply connected. One can perform the same low-energy reduction even for D-branes and talk about a D-foam background [127]. Even though these calculations cannot tell us the details of the basic constituents of spacetime, Planck-scale spacetime fluctuations are a direct result of the activities of these substructures. Since they will affect all the physics happening at lower energies, they are worthy of much closer scrutiny. It is the only hope for us earthlings confined by the shackles of low energy to fathom the blue yonder.

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REFERENCES

[1] N. D. Birrell, and P. C. Davis, *Quantum Fields in Curved Space* (Cambridge University Press, New York, 1982).

[2] Ya. Zel'dovich and A. Starobinsky, *Zh. Eksp. Teor. Fiz.* **61**, 2161 (1971) [*Sov. Phys. JETP* **34**, 1159 (1971)]; L. Grishchuk, *Ann. N. Y. Acad. Sci.* **302**, 439 (1976); B. L. Hu and L. Parker, *Phys. Lett.* **63A**, 217 (1977); B. L. Hu and L. Parker, *Phys. Rev. D* **17**, 933 (1978); F. V. Fischetti, J. B. Hartle, and B. L. Hu, *Phys. Rev. D* **20**, 1757 (1979); J. B. Hartle and B. L. Hu, *Phys. Rev. D* **20**, 1772 (1979); **21**, 2756 (1980); J. B. Hartle, *Phys. Rev. D* **23**, 2121 (1981); P. A. Anderson, *Phys. Rev. D* **28**, 271 (1983); **29**, 615 (1984).

[3] E. Calzetta and B. L. Hu, *Phys. Rev. D* **35**, 495 (1987).

[4] J. Schwinger, *J. Math. Phys.* **2**, 407 (1961), P. M. Bakshi and K. T. Mahanthappa, *J. Math. Phys.* **4**, 1, 12 (1963); L. V. Keldysh, *Zh. Eksp. Teor. Fiz.* **47**, 1515 (1964) [*Sov.

Phys. JEPT **20**, 1018 (1965)]; G. Zhou, Z. Su, B. Hao, and L. Yu, *Phys. Rep.* **118**, 1 (1985); Z. Su, L. Y. Chen, X. Yu, and K. Chou, *Phys. Rev. B* **37**, 9810 (1988); B. S. DeWitt, In *Quantum Concepts in Space and Time*, R. Penrose and C. J. Isham, eds. (Clarendon Press, Oxford, 1986); R. D. Jordan, *Phys. Rev. D* **33**, 44 (1986); E. Calzetta and B. L. Hu, *Phys. Rev. D* **35**, 495 (1987); A. Campos and E. Verdaguer, *Phys. Rev. D* **49**, 1861 (1994).

[5] B. L. Hu, *Physica A* **158**, 399 (1989).

[6] C. Kiefer, *Class. Quant. Grav.* **4**, 1369 (1987); J. J. Halliwell, *Phys. Rev. D* **39**, 2912 (1989); T. Padmanabhan, *Phys. Rev. D* **39**, 2924 (1989); B. L. Hu, Quantum and statistical effects in superspace cosmology, in *Quantum Mechanics in Curved Space time*, J. Audretsch and V. de Sabbata, eds. (Plenum, New York, 1990); E. Calzetta, *Class. Quant. Grav.* **6**, L227 (1989); *Phys. Rev. D* **43**, 2498 (1991); J. P. Paz and S. Sinha, *Phys. Rev. D* **44**, 1038 (1991); **45**, 2823 (1992); B. L. Hu, J. P. Paz, and S. Sinha, Minisuperspace as a quantum open system, in *Directions in General Relativity*, Vol. 1, B. L. Hu, M. P. Ryan, and C. V. Vishveswara, eds. (Cambridge University Press, Cambridge, 1993).

[7] E. B. Davies, *The Quantum Theory of Open Systems* (Academic Press, London, 1976); K. Lindenberg and B. J. West, *The Nonequilibrium Statistical Mechanics of Open and Closed Systems* (VCH Press, New York, 1990); U. Weiss, *Quantum Dissipative Systems* (World Scientific, Singapore, 1993).

[8] R. Feynman and F. Vernon, *Ann. Phys.* (N.Y.) **24**, 118 (1963); R. Feynman and A. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw-Hill, New York, 1965); A. O. Caldeira and A. J. Leggett, *Physica* **121A**, 587 (1983); H. Grabert, P. Schramm, and G. L. Ingold, *Phys. Rep.* **168**, 115 (1988); B. L. Hu, J. P. Paz, and Y. Zhang, *Phys. Rev. D* **45**, 2843 (1902); **47**, 1576 (1993).

[9] A. O. Caldeira and A. J. Leggett, *Physica* **121A**, 587 (1983); *Ann. Phys.* (N.Y.) **149**, 374 (1983); H. Grabert, P. Schramm, and G. L. Ingold, *Phys. Rep.* **168**, 115 (1988); B. L. Hu, J. P. Paz, and Y. Zhang, *Phys. Rev. D* **45**, 2843 (1992); **47**, 1576 (1993).

[10] B. L. Hu, Statistical mechanics and quantum cosmology, in *Proceedings Second International Workshop on Thermal Fields and Their Applications*, H. Ezawa *et al.*, eds. (North-Holland, Amsterdam, 1991).

[11] W. H. Zurek, *Phys. Rev. D* **24**, 1516 (1981); **26**, 1862 (1982); in *Frontiers of Nonequilibrium Statistical Physics*, G. T. Moore and M. O. Scully, eds. (Plenum, New York, 1986); *Physics Today* **44**, 36 (1991); E. Joos and H. D. Zeh, *Z. Phys. B* **59**, 223 (1985); A. O. Caldeira and A. J. Leggett, *Phys. Rev. A* **31**, 1059 (1985); W. G. Unruh and W. H. Zurek, *Phys. Rev. D* **40**, 1071 (1989); B. L. Hu, J. P. Paz, and Y. Zhang, *Phys. Rev. D* **45**, 2843 (1992); W. H. Zurek, *Prog. Theor. Phys.* **89**, 281 (1993); D. Giulini *et al.*, *Decoherence and the Appearance of a Classical World in Quantum Theory* (Springer-Verlag, Berlin, 1996).

[12] R. B. Griffiths, *J. Stat. Phys.* **36**, 219 (1984); R. Omnés, *J. Stat. Phys.* **53**, 893, 933, 957 (1988); *Ann. Phys.* (N.Y.) **201**, 354 (1990); *Rev. Mod. Phys.* **64**, 339 (1992); *The Interpretation of Quantum Mechanics* (Princeton University Press, Princeton, New Jersey, 1994); M. Gell-Mann and J. B. Hartle, In *Complexity, Entropy and the Physics of Information* , W. H. Zurek, ed. (Addison-Wesley, Reading, Massachusetts, 1990); *Phys. Rev. D* **47**, 3345 (1993); J. B. Hartle, Quantum mechanics of closed systems, in *Directions in General Relativity*, Vol. 1, B. L. Hu, M. P. Ryan, and C. V. Vishveswara, eds. (Cambridge University Press, Cambridge, 1993); H. F. Dowker and J. J. Halliwell, *Phys. Rev. D* **46**, 1580 (1992); T. Brun, *Phys. Rev. D* **47**, 3383 (1993); J. P. Paz and W. H. Zurek, *Phys. Rev. D* **48**, 2728 (1993); J. Twamley, *Phys. Rev. D* **48**, 5730 (1993); F. Dowker and A. Kent, *Phys. Rev. Lett.* **75**, 3038 (1995); *J. Stat. Phys.* **82**, 1575 (1996); A. Kent, *Phys. Rev. A* **54**, 4670 (1996); *Phys. Rev. Lett.* **78**, 2874 (1997); *Phys. Rev. Lett.* **81**, 1982 (1998).

[13] W. Bernard and H. B. Callen, *Rev. Mod. Phys.* **31**, 1017 (1959); R. Kubo, *Rep. Prog. Phys.* **29**, 255 (1966); L. Landau, E. Lifshitz, and L. Pitaevsky, *Statistical Physics*, Vol. 1 (Pergamon, London, 1980); R. Kubo, M. Toda, and N. Hashitsume, *Statistical Physics II* (Springer-Verlag, Berlin, 1985).
[14] R. Kubo, M. Toda, and N. Hashitsume, *Statistical Physics*, Vol. 2 (Springer-Verlag, Berlin, 1991), Chapter 4.
[15] M. Gell-Mann and J. B. Hartle, *Phys. Rev. D* **47**, 3345 (1993).
[16] Y. Zhang, Ph.D. Thesis, University of Maryland (1991); B. L. Hu, J. P. Paz, and Y. Zhang, Quantum origin of noise and fluctuation in cosmology, in *The Origin of Structure in the Universe*, E. Gunzig and P. Nardone, eds. (Plenum Press, New York, 1993), p. 227.
[17] B. L. Hu and A. Matacz, *Phys. Rev. D* **49**, 6612 (1994).
[18] E. Calzetta and B. L. Hu, *Phys. Rev D* **49**, 6636 (1994).
[19] Z. Su, L. Y. Chen, X. Yu, and K. Chou, *Phys. Rev. B* **37**, 9810 (1988).
[20] H. F. Dowker and J. J. Halliwell, *Phys. Rev. D* **46**, 1580 (1992); J. P. Paz and W. H. Zurek, *Phys. Rev. D* **48**, 2728 (1993).
[21] E. Calzetta and B. L. Hu, Decoherence of correlation histories, in *Directions in General Relativity, Vol. II: Brill Festschrift*, B. L. Hu and T. A. Jacobson, eds. (Cambridge University Press, Cambridge, 1993).
[22] A. Campos and E. Verdaguer, *Phys. Rev. D* **49**, 1861 (1994).
[23] R. Martin and E. Verdaguer, *Int. J. Theor. Phys.*, this issue; gr-qc/9812063, 9811070.
[24] B. L. Hu, In *Proceedings of the Third International Workshop on Thermal Fields and Its Applications*, R. Kobes and G. Kunstatter, eds. (World Scientific, Singapore, 1994).
[25] B. L. Hu and S. Sinha, *Phys. Rev. D* **51**, 1587 (1995).
[26] B. L. Hu and A. Matacz, *Phys. Rev. D* **51**, 1577 (1995).
[27] A. Campos and E. Verdaguer, *Phys. Rev. D* **53**, 1927 (1996).
[28] S. A. Fulling and L. Parker, *Ann. Phys.* (N.Y.) **87**, 176 (1974).
[29] E. Calzetta, A. Campos, and E. Verdaguer, *Phys. Rev. D* **56**, 2163 (1997).
[30] G. Horowitz, *Phys. Rev. D* **21**, 1445 (1980).
[31] F. C. Lombardo and F. D. Mazzitelli, *Phys. Rev. D.* **55**, 3889 (1997).
[32] F. C. Lombardo and F. D. Mazzitelli, *Phys. Rev. D.* **58**, 024009 (1998); F. C. Lombardo, F. D. Mazzitelli, and J. Russo, *Phys. Rev. D.* **59**, 064007 (1999).
[33] P. Johnson and B. L. Hu, Quantum stochastic theory of relativistic particle–field interaction: Effect of strong fields on the trajectory of a particle detector, in preparation.
[34] E. Calzetta and B. L. Hu, Correlations, decoherence, disspation and noise in quantum field theory, in *Heat Kernel Techniques and Quantum Gravity*, S. Fulling, ed. (Texas A&M Press, College Station, Texas, 1995); hep-th/9501040.
[35] E. Calzetta and B. L. Hu, Nonequilibrium quantum fields: Master effective action, correlation noise and stochastic Boltzmann equation, in preparation.
[36] B. L. Hu, Alpan Raval, and S. Sinha, Notes on black hole fluctuations and backreaction, in *Black Holes, Gravitational Radiation and the Universe*, B. R. Iyer and B. Bhawal, eds. (Kluwer, Dordtrecht, 1999).
[37] N. G. Phillips, Ph.D. Thesis, University of Maryland (1999).
[38] B. L. Hu, N. Phillips, and A. Raval, Fluctuations of the energy momentum tensor of a quantum field in a black hole spacetime, in preparation.
[39] B. L. Hu, Alpan Raval, and S. Sinha, Backreaction of a radiating quantum black hole and fluctuation-dissipation relation, in preparation.
[40] E. Calzetta and E. Verdaguer, *Phys. Rev. D* **59**, 083513 (1999).
[41] A. Campos and B. L. Hu, *Phys. Rev. D* **58**, 125021 (1998).

[42] J. A. Wheeler, *Ann. Phys.* (N.Y.) **2**, 604 (1957); *Geometrodynamics* (Academic Press, London, 1962); in *Relativity, Groups and Topology*, B. and C. DeWitt, eds. (Gordon and Breach, New York, 1964).
[43] L. H. Ford, *Phys. Rev. D* **51**, 1692 (1995).
[44] C.-I. Kuo and L. H. Ford, *Phys. Rev. D* **47**, 4510 (1993).
[45] N. Phillips and B. L. Hu, *Phys. Rev. D* **55**, 6123 (1997).
[46] E. Calzetta and B.-L. Hu, *Phys. Rev. D* **52**, 6770 (1995); E. Calzetta and S. Gonorazky, *Phys. Rev. D* **55**, 1812 (1997); A. Matacz, *Phys. Rev. D* **55**, 1860 (1997); **56**, R1836 (1997).
[47] B. S. DeWitt, *Dynamical Theory of Groups and Fields* (Gordon and Breach, 1965); *Phys. Rep. C* **19**, 295 (1975); S. Christensen, *Phys. Rev. D* **14**, 2490 (1976).
[48] P. R. Anderson, W. A. Hiscock, and D. A. Samuel, *Phys. Rev. Lett.* **70**, 1739 (1993); *Phys. Rev. D* **51**, 4337 (1995).
[49] D. N. Page, *Phys. Rev. D* **25**, 1499 (1982).
[50] L. H. Ford, Cosmological and black hole horizon fluctuations, gr-qc/9704050.
[51] A. Casher, F. Englert, N. Itzhaki, and R. Parentani, Black hole horizon fluctuations, hep-th/9606106.
[52] R. D. Sorkin, How wrinkled is the surface of a black hole?, gr-qc/9701056.
[53] C. Barrabès, V. Frolov, and R. Parentani, Metric fluctuation correction to Hawking radiation, gr-qc/9812076.
[54] P. Anderson *et al.*, In *Heat Kernei Techniques and Quantum Gravity*, S. A. Fulling, ed. (Texas A&M University Press, College Station, Texas, 1995).
[55] J. W. York, Jr., *Phys. Rev. D* **28**, 2929 (1983); **31**, 775 (1985); **33**, 2092 (1986).
[56] J. M. Bardeen, *Phys. Rev. Lett.* **46**, 382 (1981); P. Hajicek and W. Israel, *Phys. Lett.* **80A**, 9 (1980).
[57] S. Massar, *Phys. Rev. D* **52**, 5857 (1995).
[58] P. Candelas and D. W. Sciama, *Phys. Rev. Lett.* **38**, 1372 (1977)
[59] E. Mottola, *Phys. Rev. D* **33**, 2136 (1986)
[60] A. Campos and B. L. Hu, *Int. J. Theor. Phys.* **38**, 1253 (1999).
[61] D. J. Gross, M. J. Perry, and L. G. Yaffe, *Phys. Rev. D* **25**, 330 (1982).
[62] A. Rebhan, *Nucl. Phys. B* **351**, 706 (1991).
[63] A. P. de Almeida, F. T. Brandt, and J. Frenkel, *Phys. Rev. D* **49**, 4196 (1994); F. T. Brandt and J. Frenkel, *Phys. Rev. D* **58**, 085012 (1998), and references therein.
[64] B. L. Hu, In *Quantum Classical Correspondence*, D. S. Feng and B. L. Hu, eds. (International Press, Boston, 1997).
[65] A. D. Sakharov, Vacuum quantum fluctuations in curved space and the theory of gravitation, *Doklady Akad. Nauk SSR* **177**, 70–71 (1987) [*Sov. Phys. Doklady* **12**, 1040–1041 (1968)]; see also S. L. Adler, *Rev. Mod. Phys.* **54**, 729 (1982).
[66] B. L. Hu, General relativity as geometro-hydrodynamics, gr-qc/9607070.
[67] E. Calzetta and B. L. Hu, *Phys. Rev. D* **55**, 1795 (1997).
[68] B. L. Altshuler, P. A. Lee, and R. A. Webb, eds., *Mesoscopic Phenomena in Solids* (North-Holland, Amsterdam, 1991); B. K.Kramer, ed., *Quantum Coherence in Mesoscopic Systems* (Plenum Press, New York, 1991); W. P. Kirk and M. A. Reed, eds., *Nanostructures and Mesocopic Systems* (Academic Press, San Diego, 1992); Y. Imry, *Introduction to Mesoscopic Physics* (Wiley, New York, 1997).
[69] A. O Caldeira and A. J. Leggett, *Ann. Phys.* (N.Y) **149**, 374 (1993).
[70] S. Weinberg, *Quantum Field Theory* (Wiley, New York, 1996).
[71] B. L. Hu and Y. Zhang, Coarse-Graining, scaling, and inflation, University of Maryland Preprint 90-186 (1990); B. L. Hu, In *Relativity and Gravitation: Classical and Quantum*, J. C. D' Olivo *et al.*, eds. (World Scientific, Singapore, 1991).
[72] T. Jacobson, Introduction to black hole microscopy, hep-th/9510026.

[73] W. G. Unruh, *Phys. Rev. Lett.* **46**, 1351 (1981); *Phys. Rev. D* **51**, 2827 (1995); T. Jacobsen, *Phys. Rev. D* **44**, 1731 (1991); **53**, 7082 (1994).
[74] B. L. Hu, Fluctuation, dissipation and irreversibility in cosmology, in *The Physical Origin of Time-Asymmetry*, J. J. Halliwell, J. Perez-Mercader, and W. H. Zurek, eds. (Cambridge University Press, Cambridge, 1994).
[75] J. B. Hartle, R. Laflamme, and D. Marolf, *Phys. Rev. D* **51**, 7007 (1995); T. Brun and J. J. Halliwell, *Phys. Rev. D* **54**, 2899 (1996); J. J. Halliwell, *Phys. Rev. D* **58**, 105015 (1998); C. Anastopoulos, gr-qc/9805074; T. Brun and J. B. Hartle, quant-phys/9808024.
[76] C. W. Misner, K. S. Thorne, and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1971), Chapter 15.
[77] M. B. Green, J. H. Schwarz, and E. Witten, *Superstring Theory* (Cambridge University Press, Cambridge, 1990); E. Witten, *Physics Today* **49**, 24 (1996); J. Polchinsky, *Superstring Theory* (Cambridge University Press, Cambridge, 1998).
[78] A. Ashtekar, *Lectures on Non-Perturbative Canonical Gravity* (World Scientific, Singapore, 1991); in *Knot Theory and Quantum Gravity*, J. Baez, ed. (Oxford University, Press, Oxford, 1995); R. Gambini and J. Pullin, *Loops, Knots, Gauge Theories and Quantum Gravity* (Cambridge University Press, Cambridge, 1996); A. Ashtekar, K. Baez, A. Corichi, and K. Krasnov, *Phys. Rev. Lett.* **80**, 904 (1998).
[79] T. Regge, *Nuovo Cimento* **19**, 558 (1961); J. B. Hartle, *J. Math. Phys.* **26**, 804 (1985); **27**, 287 (1986); **30**, 452 (1989); H. W. Hamber, In *Critical Phenomena, Random Systems, Gauge Theories*, K. Osterwalder and R. Stora, eds. (North-Holland, Amsterdam, 1986); H. W. Hamber, *Nucl. Phys. B (Proc. Suppl.)* **20**, 728 (1991); **25A**, 150 (1992); *Phys. Rev. D* **45**, 507 (1992); *Nucl. Phys. B* **400**, 347 (1993); R. M. Williams and P. A. Tucker, *Class. Quant. Grav.* **9**, 1409 (1992); H. W. Hamber and R. M. Williams, *Phys. Rev. D* **47**, 510 (1993); *Nucl. Phys.* **415**, 463 (1994).
[80] J. D. Bekenstein, *Phys. Rev. D* **7**, 1333 (1973); S. W. Hawking, *Commun. Math. Phys.* **43**, 199 (1975).
[81] R. M. Wald, *Phys. Rev. D* **48**, R3427 (1993); T. Jacobson, G. Kang, and R. Myers, *Phys. Rev. D* **49**, 6587 (1994); T. Jacobson, Black hole entropy and induced gravity, gr-qc/9404039; M. Banados, C. Teitelboim, and J. Zanelli, *Phys. Rev. Lett.* **72**, 957 (1994); J. D. Bekenstein, Do we understand black hole entropy? in *Proceedings Seventh Marcel Grossmann Meeting* (Stanford University, 1994), gr-qc/9409015; D. M. Page, Black hole information, in *Proceedings 5th Canadian Conference on General Relativity and Relativistic Astrophysics*, R. B. Mann and R. G. McLenaghan, eds. (World Scientific, Singapore, 1994), hep-th/9305040; V. P. Frolov, D. V. Fursaev, and A. I. Zelnikov, Black hole entropy: Off-shell vs on-shell, hep-th/9512184; Black hole entropy and/induced gravity, hep-th/9607104.
[82] T. Jacobson, *Phys. Rev. Lett.* **75**, 1260 (1995).
[83] L. Susskind and J. Uglam, *Phys. Rev. D* **50**, 2700 (1994); D. Kabat, S. H. Shenker, and M. J. Strassler, *Phys. Rev. D* **52**, 7027 (1995); J. D. Bekenstein and V. F. Mukhanov, *Phys. Lett. B* **360**, 7 (1995).
[84] A. Strominger and C. Vafa, *Phys. Lett. B* **379**, 99 (1996); G. T. Horowitz, The origin of black hole entropy in string theory, in *Proceedings of the Pacific Conference on Gravitation and Cosmology* (Seoul, Korea, 1996), gr-qc/9604051; G. Horowitz and J. Polchinski, *Phys. Rev. D* **55**, 6189 (1997); J. M. Maldacena, Black holes and D-branes, *Nucl. Phys. Proc. Suppl.* **61A**, 111 (1998).
[85] A. Ashtekhar, In *String Gravity and Physics at the Planck Energy Scale*, N. Sanchez and A. Zichichi, eds. (Kluwer, Dordrecht, 1996).

[86] T. Regge, *Nuovo Cimento* **19**, 558 (1961); J. B. Hartle, *J. Math Phys.* **26**, 804 (1985); **27**, 287 (1986); **30**, 452 (1989); H. W. Hamber, In *Critical Phenomena, Random Systems, Gauge Theories* K Osterwalder and R. Stora, eds. (North-Holland, Amsterdam, 1986).
[87] G. Ponzano and T. Regge, Semiclassical limit of Racah coefficients, in *Spectroscopic and Group Theoretical Methods in Physics*, F. Bloch, ed. (North-Holland, Amsterdam, 1968); J. Iwasaki, A reformulation of the Ponzano–Regge quantum gravity models in terms of surfaces (1994); J. W. Barrett and T. J. Foxon, *Class. Quant. Grav.* **11**, 543 (1994); J. W. Barrett, Quantum gravity as topological quantum field theory (1995).
[88] D. Nelson *et al.*, eds., *Statistical Mechanics of Membranes and Surfaces* (World Scientific, Singapore, 1989); D. J. Gross, T. Piran, and S. Weinberg, eds., *Two-Dimensional Quantum Gravity and Random Surfaces* (World Scientific, Singapore, 1992); F. David, P. Ginsparg, and J. Jinn-Justin, eds. *Fluctuating Geometries in Statistical Mechanics and Field Theory* (North-Holland, Amsterdam, 1996); J. Ambjorn, M. Carfora, and A. Marzuoli, *The Geometry of Dynamical Triangulations* (Springer, Berlin, 1997).
[89] B. L. Hu, Cosmology as 'condensed matter' physics, in *Proceedings Third Asia Pacific Physics Conference*, Y. W. Chan *et al.*, eds. (World Scientific, Singapore, 1988), Vol. 1, p. 301; gr-qc/9511076.
[90] L. Susskind, *J. Math. Phys.* **36**, 6377 (1995).
[91] G. 't Hooft, Quantization of point particles in 2+1 dimensional gravity and spacetime discreteness, gr-qc/9601014.
[92] T. Appelquist and J. Carazzone, *Phys. Rev. D* **11**, 2856 (1975); S. Weinberg, *Phys. Lett.* **83B**, 339 (1979); B. Ovrut and H. J. Schnitzer, *Phys. Rev. D* **21**, 3369 (1980); **22**, 2518 (1980); L. Hall, *Nucl. Phys. B* **178**, 75 (1981); P. Lapage, In *From Actions to Answers*, T. DeGrand and D. Toussaint, eds. (World Scientific, Singapore, 1990), p. 483; S. Weinberg, *The Quantum Theory of Fields*, Vol. 1 (Cambridge University Press, Cambridge, 1995).
[93] J. B. Hartle and B. L. Hu, *Phys. Rev. D* **20**, 1772 (1979); *21*, 2756 (1980).
[94] J. Schwinger, *Phys. Rev.* **82**, 664 (1951).
[95] E. Mottola, *Phys. Rev. D* **33**, 2136 (1986).
[96] E. Calzetta and B.-L. Hu, *Phys. Rev. D* **37**, 2878 (1988).
[97] P. Danielewicz, *Ann. Phys.* (N.Y.) **152**, 239 (1984); S-P. Li and L. McLerran, *Nucl. Phys. B* **214**, 417 (1983).
[98] L. P. Kadanoff and G. Baym, *Quantum Statistical Mechanics* (Benjamin, New York, 1962); D. F. Du Bois, In *Lectures in Theoretical Physics*, Vol 9c, W. Brittin, ed. (Gordon and Breach, New York, 1967); B. Bezzerides and D. F. Du Bois, *Ann. Phys.* (N.Y.) **70**, 10 (1972).
[99] St. Mrowczynski and P. Danielewicz, *Nucl. Phys. B* **342**, 345 (1990); St. Mrowczynski and U. Heinz, *Ann. Phys.* **229**, 1 (1994); P. Henning, *Phys. Rep.* **253**, 235 (1995); *Nucl. Phys. A* **582**, 633 (1995); D. Boyanovsky, I. Lawrie, and D.-S. Lee, *Phys. Rev. D* **54**, 4013 (1996); E. Wang and U. Heinz, *Phys. Rev. D* **53**, 899 (1996).
[100] M. Kac and J. Logan, Fluctuations, in *Fluctuation Phenomena*, E. W. Montroll and J. L. Lebowitz, eds. (Elsevier, New York, 1979), p. 1.
[101] R. Jackiw, *Phys. Rev. D* **9**, 1686 (1974); J. Iliopoulos, C. Itzykson, and A. Martin, *Rev. Mod. Phys.* **47**, 165 (1975).
[102] H. D. Dahmen and G. Jona-Lasinio, *Nuovo Cimento* **52A**, 807 (1962); C. de Dominicis and P. Martin, *J. Math. Phys.* **5**, 14 (1964); J. M. Cornwall, R. Jackiw, and E. Tomboulis, *Phys. Rev. D* **10**, 2428 (1974); R. E. Norton and J. M. Cornwall, *Ann. Phys.* (N.Y.) **91**, 106 (1975).
[103] J. B. Hartle and G. Horowitz, *Phys. Rev. D* **24**,, 257 (1981).
[104] Y. Kluger, E. Mottola, and J. M. Eisenberg, *Phys. Rev. D* **58**, 125015 (1998).

[105] F. A. Buot, *Phys. Rep.* **234**, 73–174 (1993); S. Datta, *Electronic Transport in Mesoscopic Systems* (Cambridge University Press, Cambridge, 1995); P. Sheng, *Introduction to Wave Scattering, Localization and Mesoscopic Phenomena* (Academic Press, New York, 1995).
[106] S. Chandrasekhar and B. C. Xanthopoulos, *Proc. Roy. Soc. A* **408**, 175 (1986); **410**, 311 (1987); A. G. Mirzbekian and G. Vilkovisky, gr-qc/9803006.
[107] G. Stephens, E. Calzetta, B. L. Hu, and S. A. Ramsey, *Phys. Rev. D* **59**, 045009 (1999).
[108] V. A. Rubakov, *Nucl. Phys. B* **245**, 481 (1991).
[109] T. Vachaspati and A. Vilenkin, *Phys. Rev. D* **37**, 898 (1988).
[110] S. A. Ramsey and B. L. Hu, *Phys. Rev. D* **56**, 678 (1997).
[111] A. O. Caldeira and A. J. Leggett, *Ann. Phys.* (N.Y.) **149**, 374 (1983).
[112] A. Vilenkin, *Phys. Rev. D* **27**, 2848 (1983); **30**, 509 (1984); *Phys. Lett.* **117B**, 25 (1985); *Nucl. Phys. B* **226**, 527 (1983).
[113] S. W. Hawking, *Comm. Math. Phys.* **87**, 395 (1982).
[114] D. N. Page, *Phys. Rev. D* **34**, 2267 (1986).
[115] P. Ginzparg and M. J. Perry, *Nucl. Phys. B* **222**, 245 (1983).
[116] B. F. Whiting and J. W. York, Jr., *Phys. Rev. Lett.* **61**, 1336 (1988).
[117] R. K. P. Zia, Driven diffusive system, in *Phase Transitions and Critical Phenomena*, Vol. 20, C. Domb and J. Lebowitz, eds. (Academic Press, New York, 1995).
[118] M. Kadar, G. Parisi, and Y. C. Zhang, *Phys. Rev. Lett.* **56**, 342 (1986).
[119] B. L. Hu and Y. Zhang, *Mod. Phys. Lett. A* **8**, 3575 (1993); *Int. J. Mod. Phys.* **10**, 4537 (1995).
[120] A. Anderson and J. J. Halliwell, *Phys. Rev D* **48**, 2753 (1993); A. Anastopoulos and J. J. Halliwell, *Phys. Rev. D.* **51**, 6870 (1995).
[121] B. L. Hu and K. Shiokawa, *Phys. Rev. D* **57**, 3474 (1998).
[122] S. W. Hawking, *Nucl. Phys. B* **144**, 349 (1978); S. W. Hawking, D. N. Page, and C. N. Pope, *Nucl. Phys. B* **170**[FS1], 283 (1980); N. P. Warner, *Comm. Math. Phys.* **86**, 419 (1982).
[123] S. W. Hawking, *Phys. Rev. D* **37**, 904 (1988); S. Coleman, *Nucl. Phys. B* **307**, 867 (1988); S. Coleman, J. B. Hartle, T. Piran, and S. Weinberg, eds., *Quantum Cosmology and Baby Universes* (World Scientific, Singapore, 1991)
[124] S. W. Hawking, *Phys. Rev. D* **53**, 3099 (1996).
[125] D. Garfunkle, S. B. Giddings, and A. Stominger, *Phys. Rev. D* **49**, 958 (1994); F. Dowker *et al., Phys. Rev. D* **50**, 2662 (1994); S. W. Hawking, G. T. Horowitz, and S. F. Ross, *Phys. Rev. D* **51**, 4302 (1995); R. Busso and S. W. Hawking, *Phys. Rev.* **52**, 5659 (1995); **54**, 6312 (1996).
[126] S. Carlip, *Phys. Rev. Lett.* **79**, 4071 (1997); *Class. Quant. Grav.* **15**, 2629 (1998); L. J. Garay, *Phys. Rev. Lett.* **80**, 2508 (1998); *Phys. Rev. D* **58**, 124015 (1998).
[127] J. Ellis, N. E. Mavromatos, and D. V. Nanopoulos, Quantum decoherence in a D-foam background, *Mod. Phys. Lett A.* (1997).
[128] K. Huang, *Statistical Mechanics*, 2nd ed. (Wiley, New York, 1987).
[129] A. I. Akhiezer and S. V. Peletminsky, *Methods of Statistical Physics* (Pergamon, London, 1981).